Risk-adjusted Performance Measurement

"Alpha to Omega"

"Downside to Drawdown"

"Appraisal to Pain"

Carl Bacon

Zurich

6th February 2008

Risk Management

- **Risk Managers**
  - Front Office
  - Paid to take risk
  - Risk is good
- **Risk Controllers**
  - Middle office
  - Paid to monitor/reduce risk
  - Risk is bad

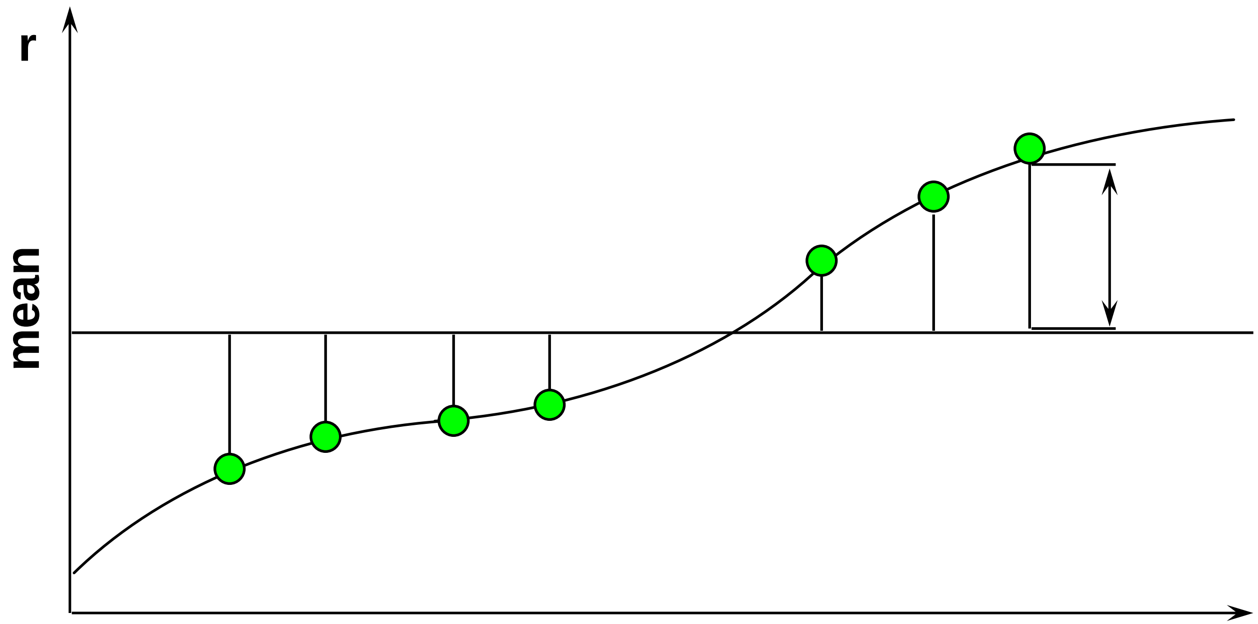
Risk Measures

- Ex-Post
  - Risk after the event
  - Historical
- Ex-Ante
  - Risk before the event
  - Prospective

Simple Risk Measures

- Absolute
  - Variability, Sharpe Ratio
- Relative
  - Tracking Error, Information Ratio
- Regression
  - α , β

Variability



$$\sigma = \sqrt{\frac{\Sigma[r_i\text{-mean}]^2}{n}}$$

Standard Deviation

- Variance

$$\sigma^2 = \frac{\sum (r_i - mean)^2}{n}$$

- Standard deviation

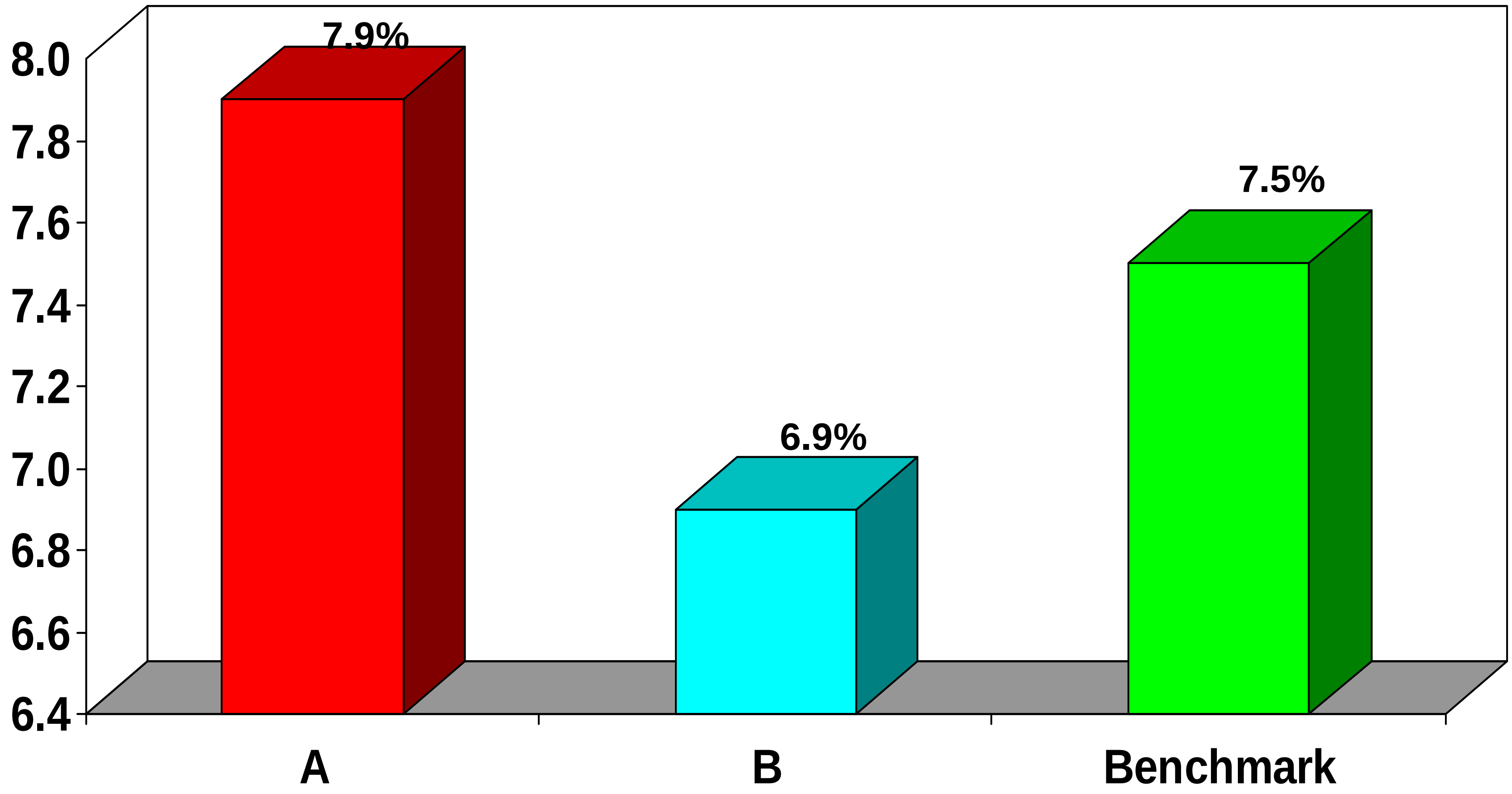
$$\sigma = \sqrt{\frac{\sum (r_i - mean)^2}{n}}$$

Note annualised standard deviation $\sigma^A = \sqrt{t} \times \sigma$

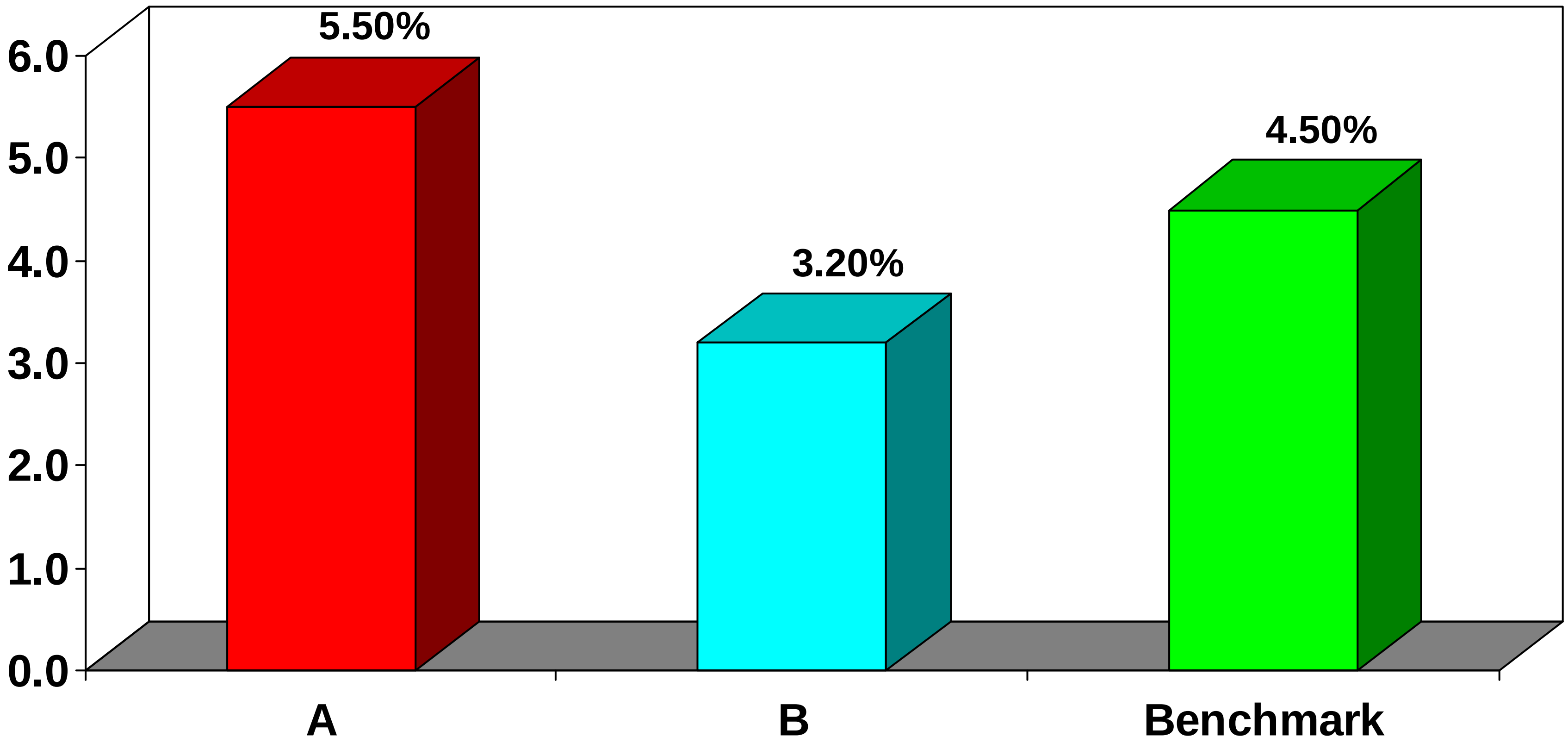
t=frequency of observations in year

(Central Limit Theorem)

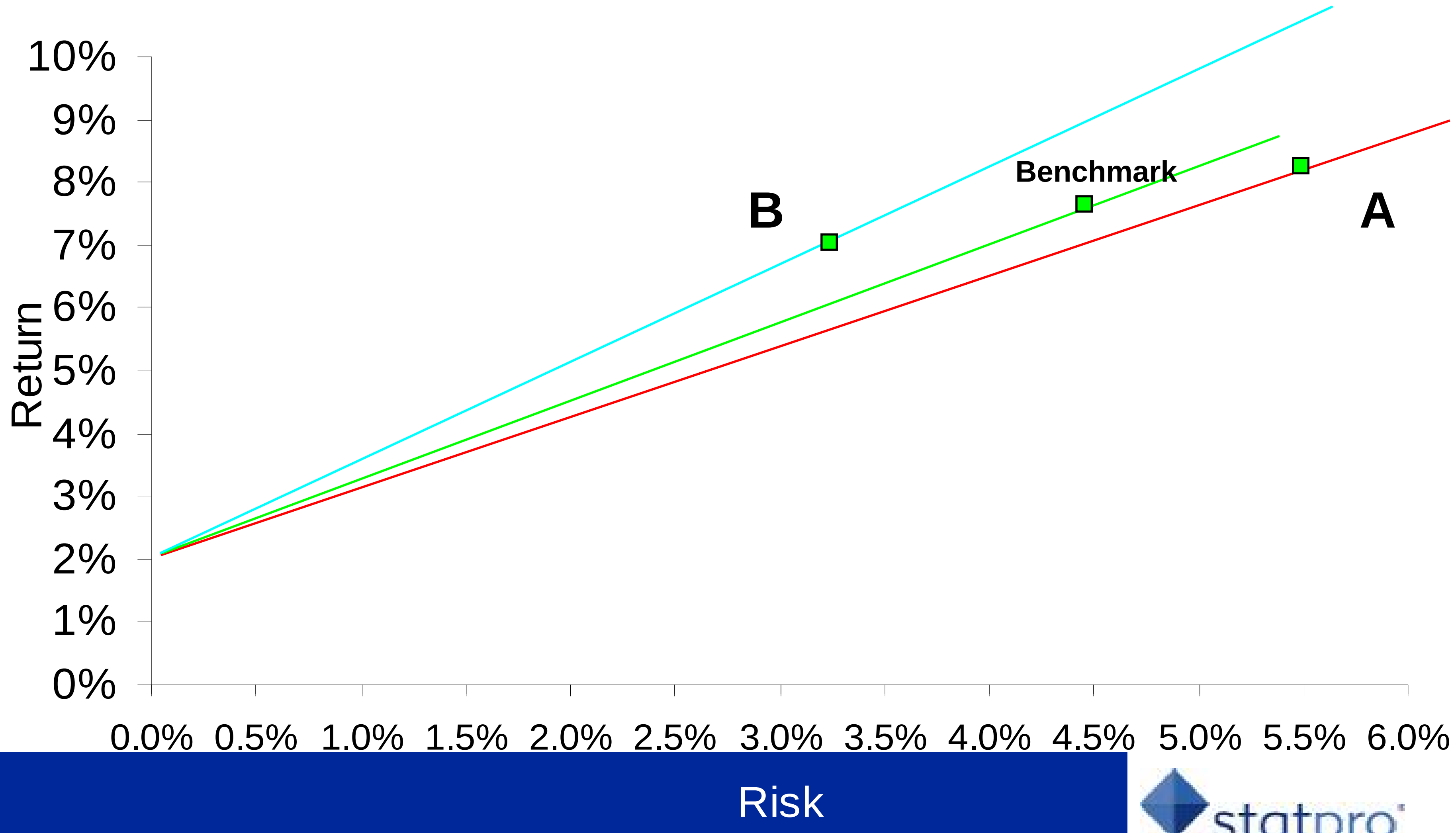
Portfolio Returns



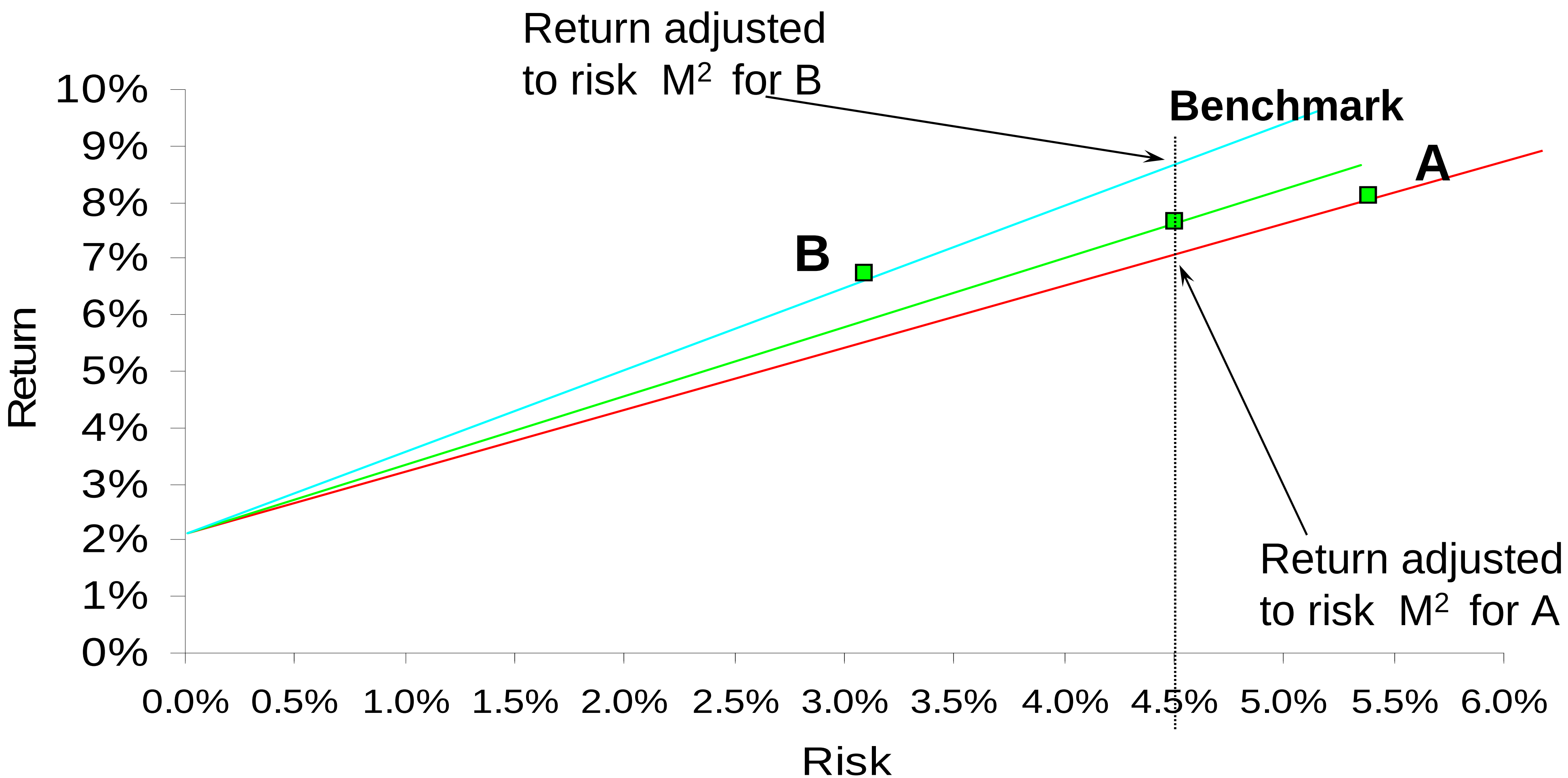
Portfolio Risk



Sharpe Ratio



M²


Return adjusted
to risk M2 for B
Benchmark
A
B
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
Return
0.0% 0.5% 1.0% 1.5% 2.0% 2.5% 3.0% 3.5% 4.0% 4.5% 5.0% 5.5% 6.0%
Risk
Return adjusted
to risk M2 for A

Risk Adjusted Returns

Sharpe Ratio

$$SR = \frac{r_P - r_F}{\sigma_P}$$

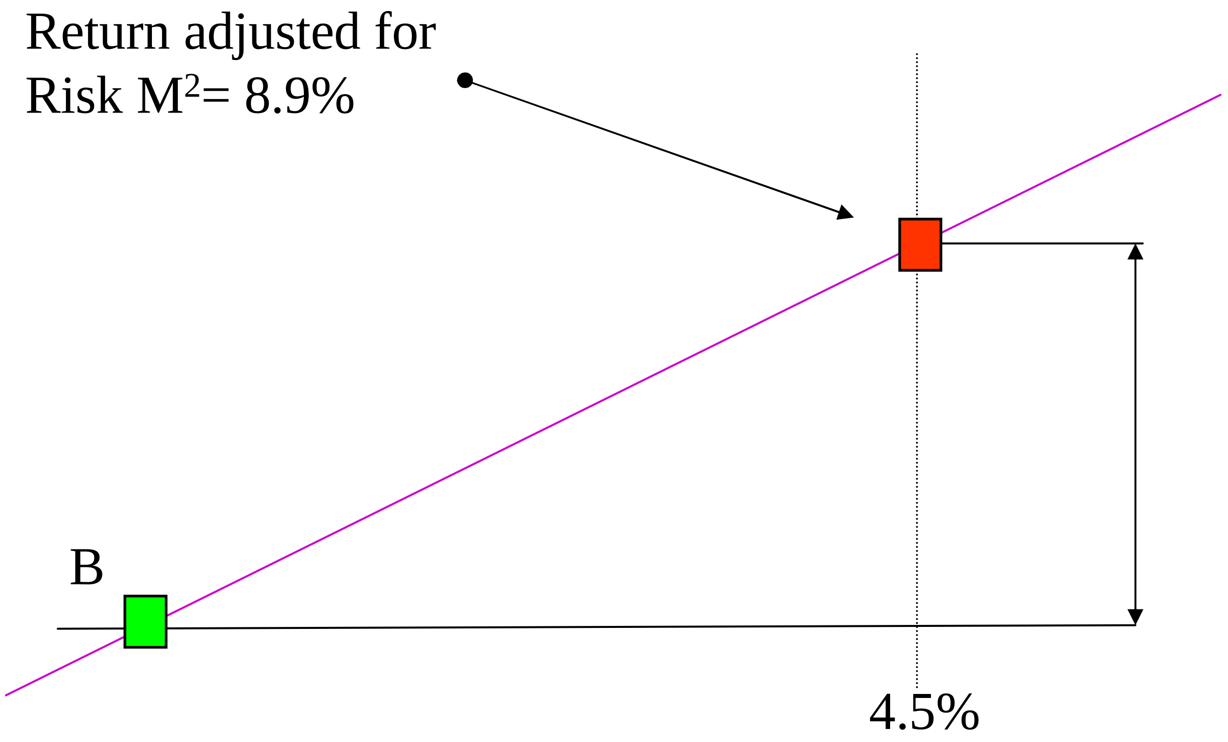
$$M^2 = r_P + SR \times (\sigma_M - \sigma_p)$$

or

$$M^2 = (r_P - r_F) \times \frac{\sigma_M}{\sigma_P} + r_F$$

σ_P = Portfolio Risk
σ_M = Market Risk
R_P = Portfolio Return
R_F = Risk Free Rate

Modigliani Excess Return



M^2 (Excess) =8.9% - 7.5% = 1.4%

Risk –adjusted excess return

Tracking error

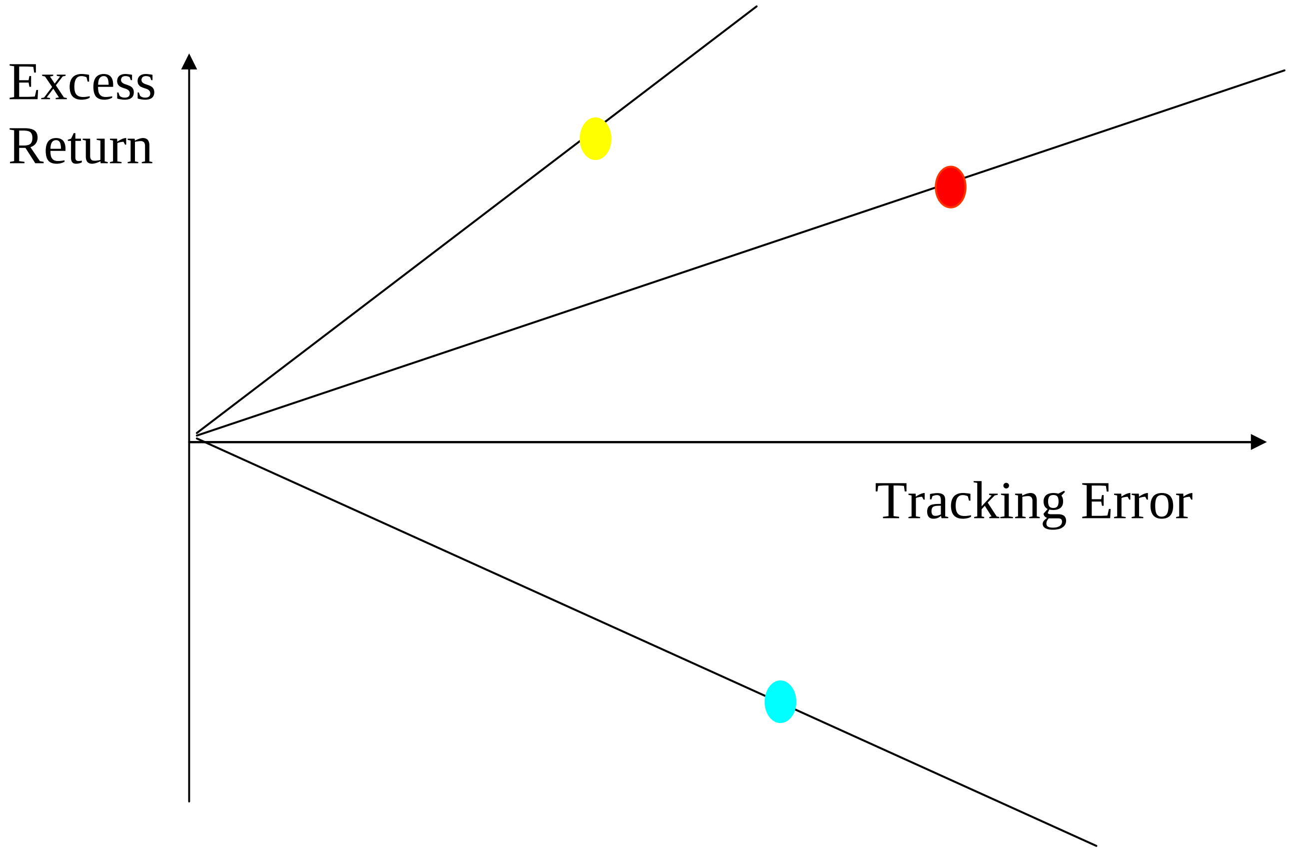
- Tracking Error
  - Standard Deviation of excess return

- Measures consistency of excess return
  - Ex-Post or Ex-ante
  - Normally annualised

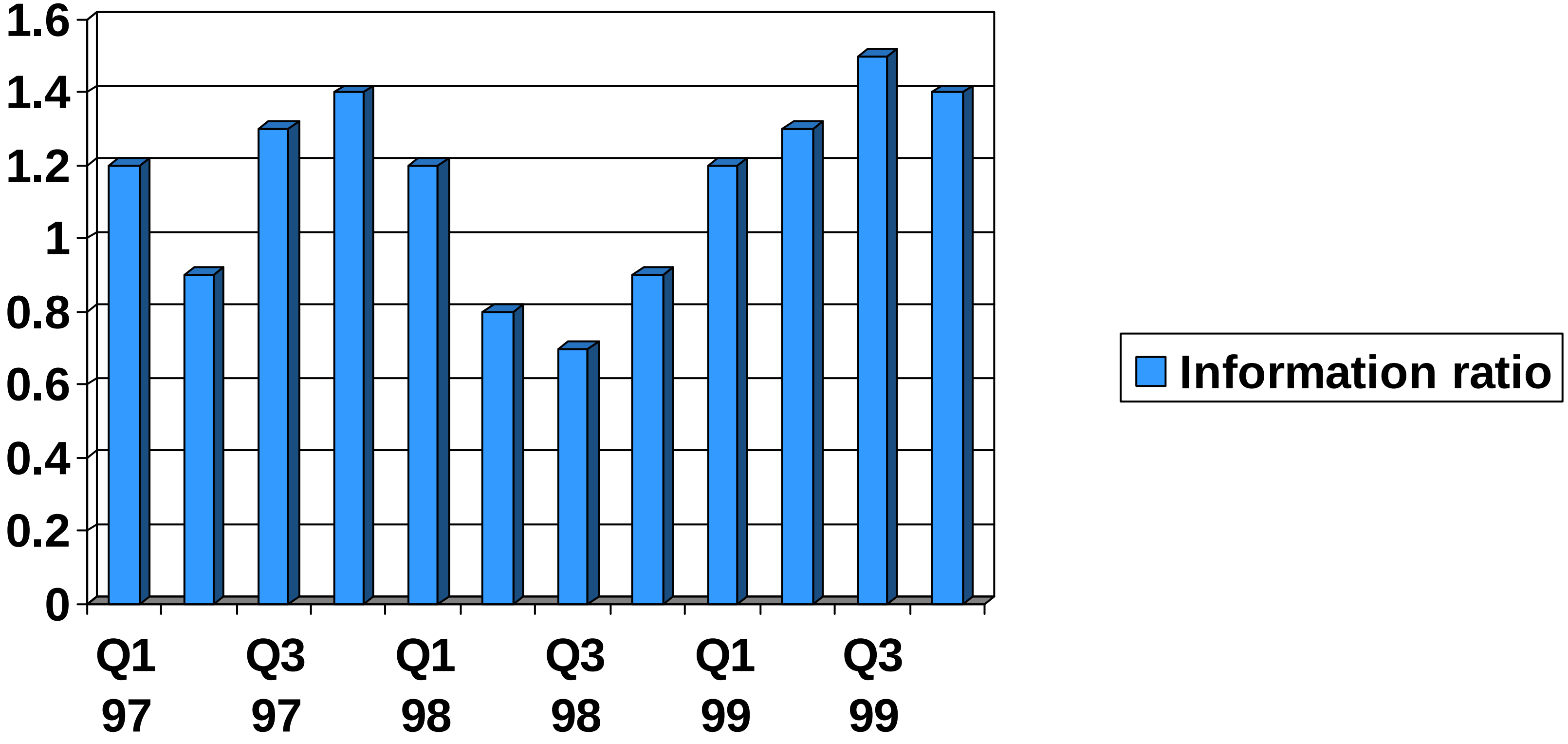
- Information Ratio

$$\frac{\text{Annualised Excess Return}}{\text{Annualised Tracking Error}}$$

Information Ratio



Information Ratio



Risk Efficiency Ratio

Risk Efficiency Ratio

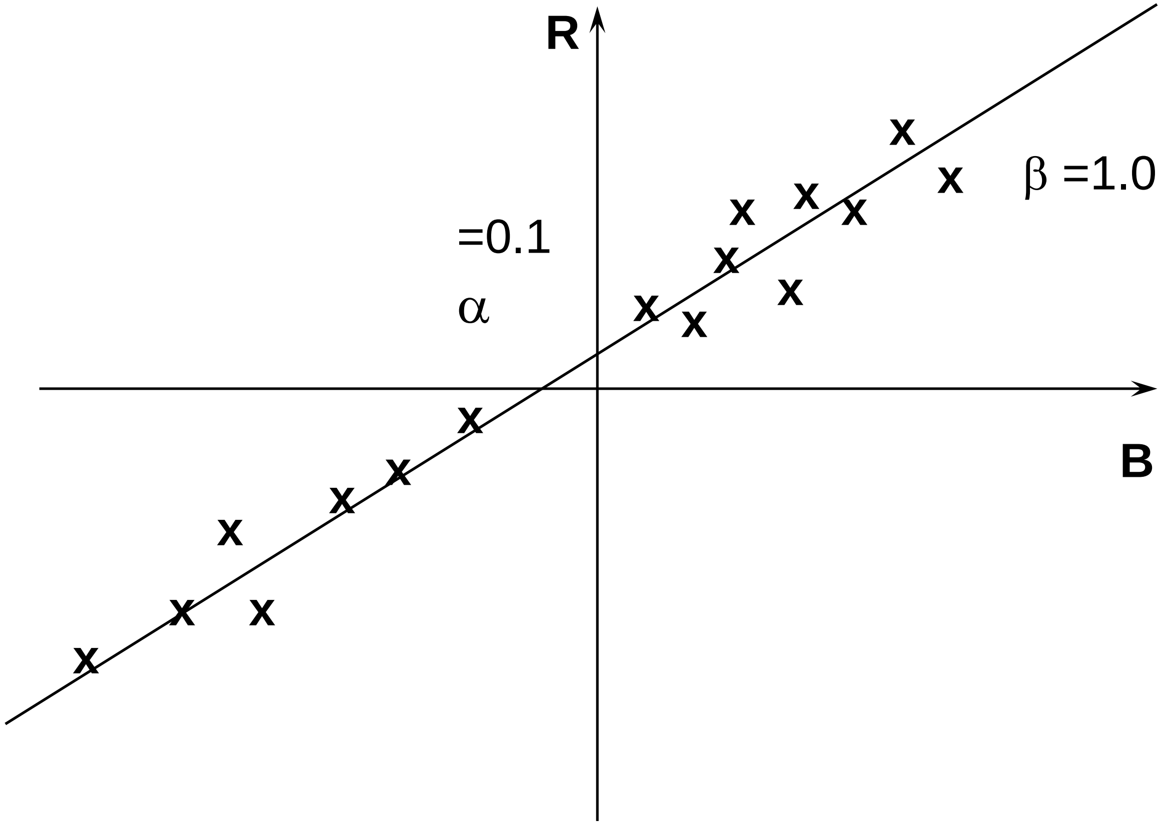
$$\frac{\text{Observed Tracking Error}}{\text{Forecast Tracking Error}}$$

or

$$\frac{\text{Ex-Post Tracking Error}}{\text{Ex-Ante Tracking Error}}$$

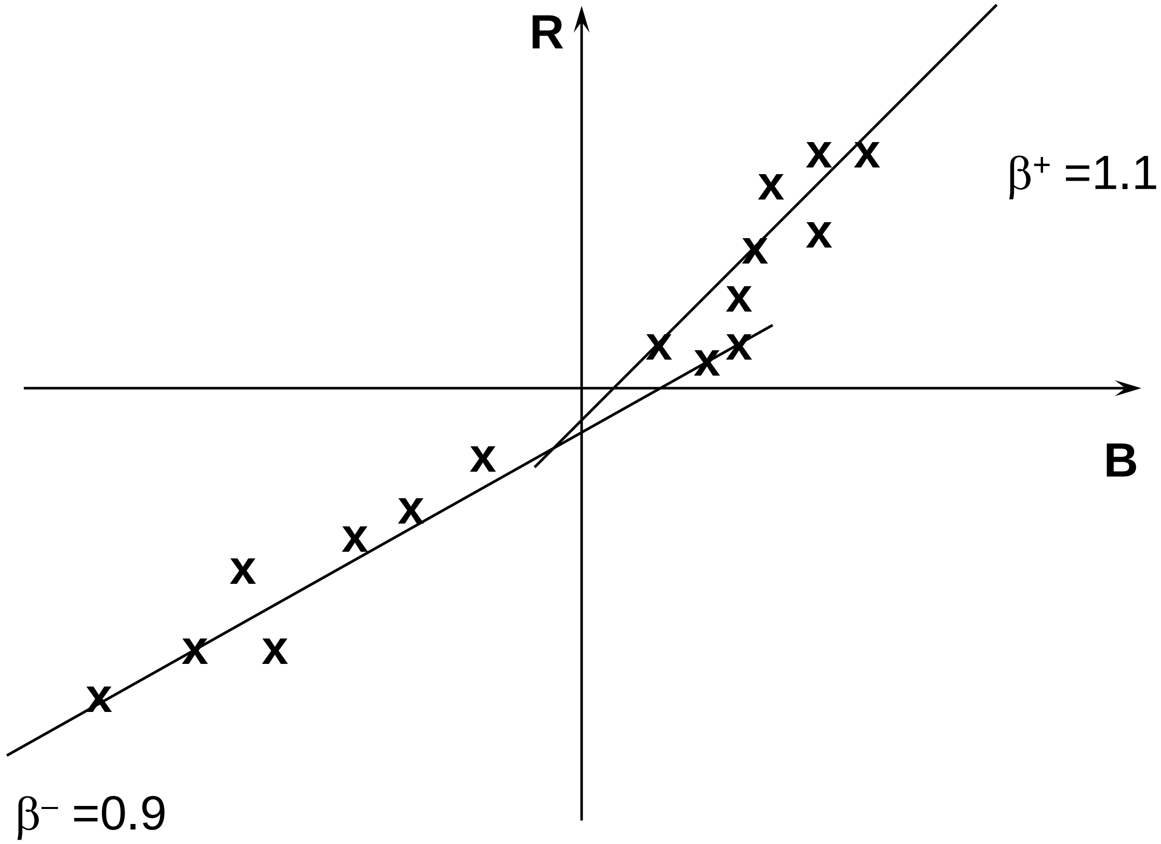
Measures the quality of the tracking error forecast

Regression Equation



$$R = \alpha + \beta\,(B - R_F) + \varepsilon$$

Bull & Bear Betas



Regression Equation

Regression Equation $r_p - r_F = \alpha + \beta(r_M - r_F) + \varepsilon$

Beta (β) Gradient of the regression equation

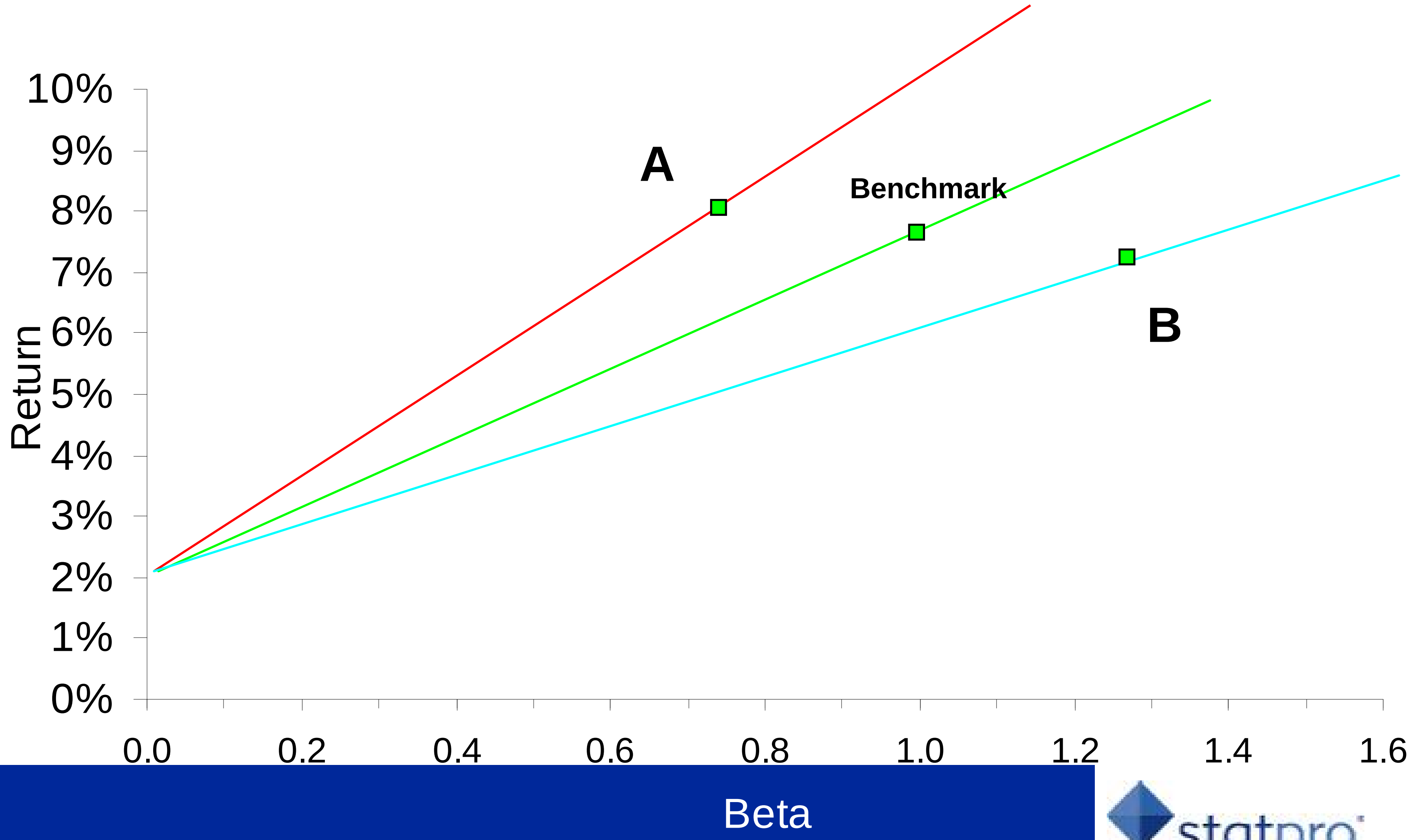
Jensen's Alpha (α) Intercept of the Regression Equation

Regression Statistics

- Correlation (r *or* ρ) = $\dfrac{\text{Systematic Risk}}{\text{Portfolio Risk}}$

- Coefficient of Determination (R^2) = $\dfrac{\text{Systematic Variance}}{\text{Portfolio Variance}}$

Treynor Ratio



Risk Measures

Treynor Ratio

$$TR = \frac{r_P - r_F}{\beta}$$

Systematic Risk

$$\sigma_s = \beta \times \sigma_m$$

Appraisal Ratio

$$\frac{\text{Jensen's Alpha}}{\text{Specific Risk}} = \frac{\alpha}{\sigma_\varepsilon}$$

Modified Jensen

$$\frac{\alpha}{\beta}$$

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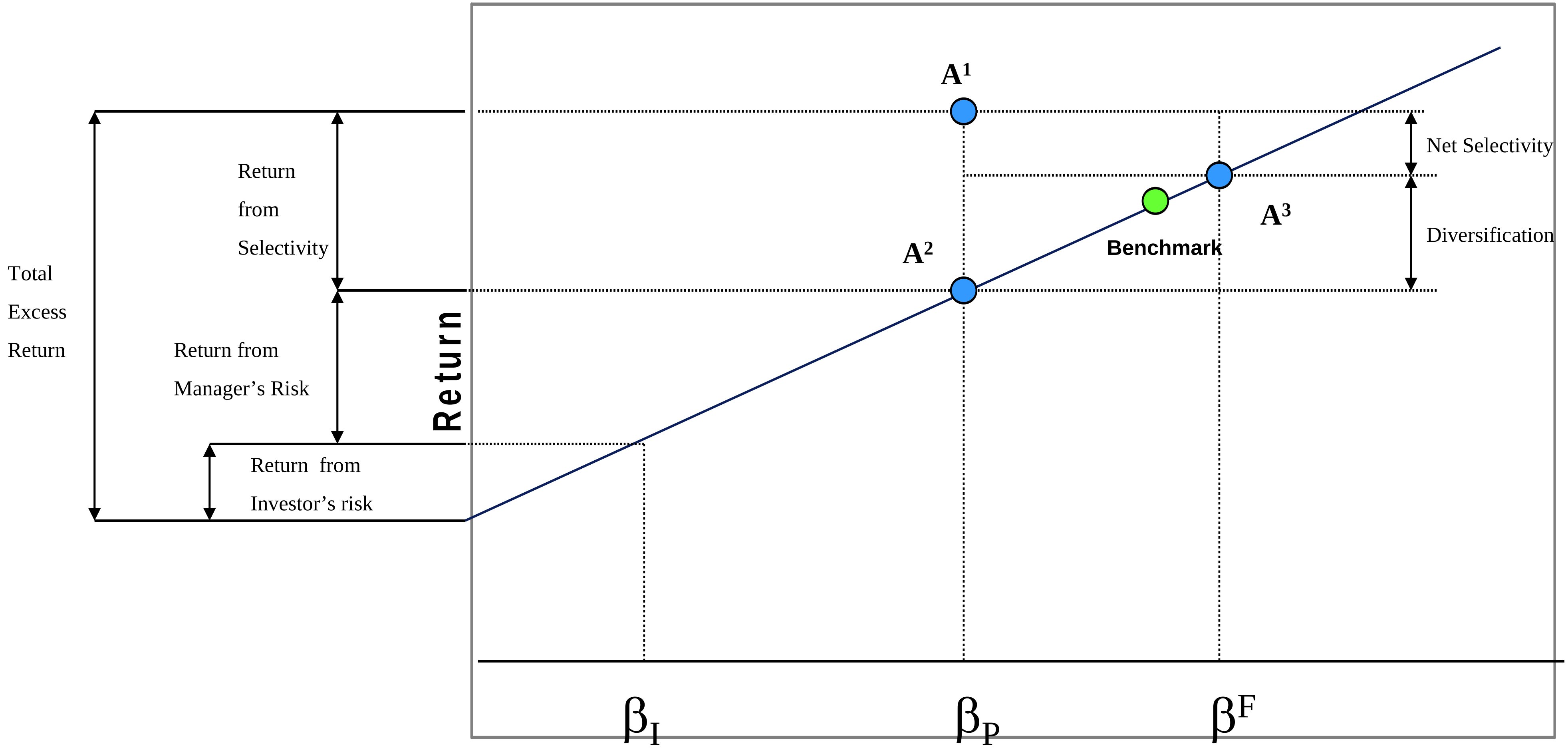
Fama Decomposition

- Early form of performance attribution
- Return above risk free rate decomposed into two components:
  - Systematic Risk
    - Investor’s risk
    - Manager’s risk
  - Selectivity (Jensen’s alpha)
    - Diversification
    - Net selectivity

Fama Decomposition



Fama Decomposition: Systematic risk

Return due to systematic risk

$$r_{\beta_p} = \beta_P \times (r_M - r_F)$$

Return due to investor's systematic risk

$$r_{\beta_I} = \beta_I \times (r_M - r_F)$$

Return due to manager's systematic risk

$$r_{\beta_M} = (\beta_P - \beta_I) \times (r_M - r_F)$$

Fama Decomposition

Fama Equivalent β^F

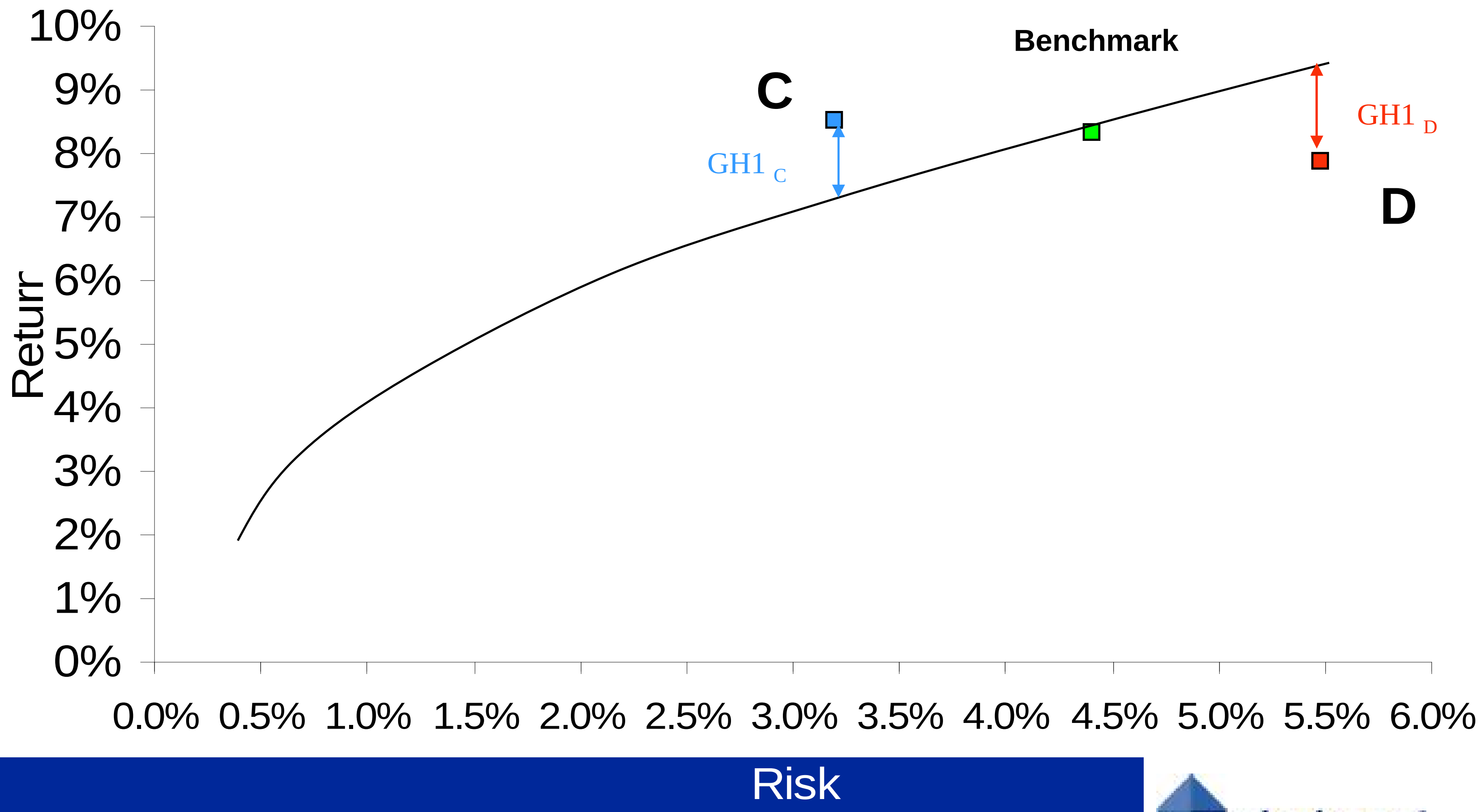
$$\beta^F = \frac{\sigma_P}{\sigma_M}$$

Diversification (d) *(Return required to justified specific risk taken)*

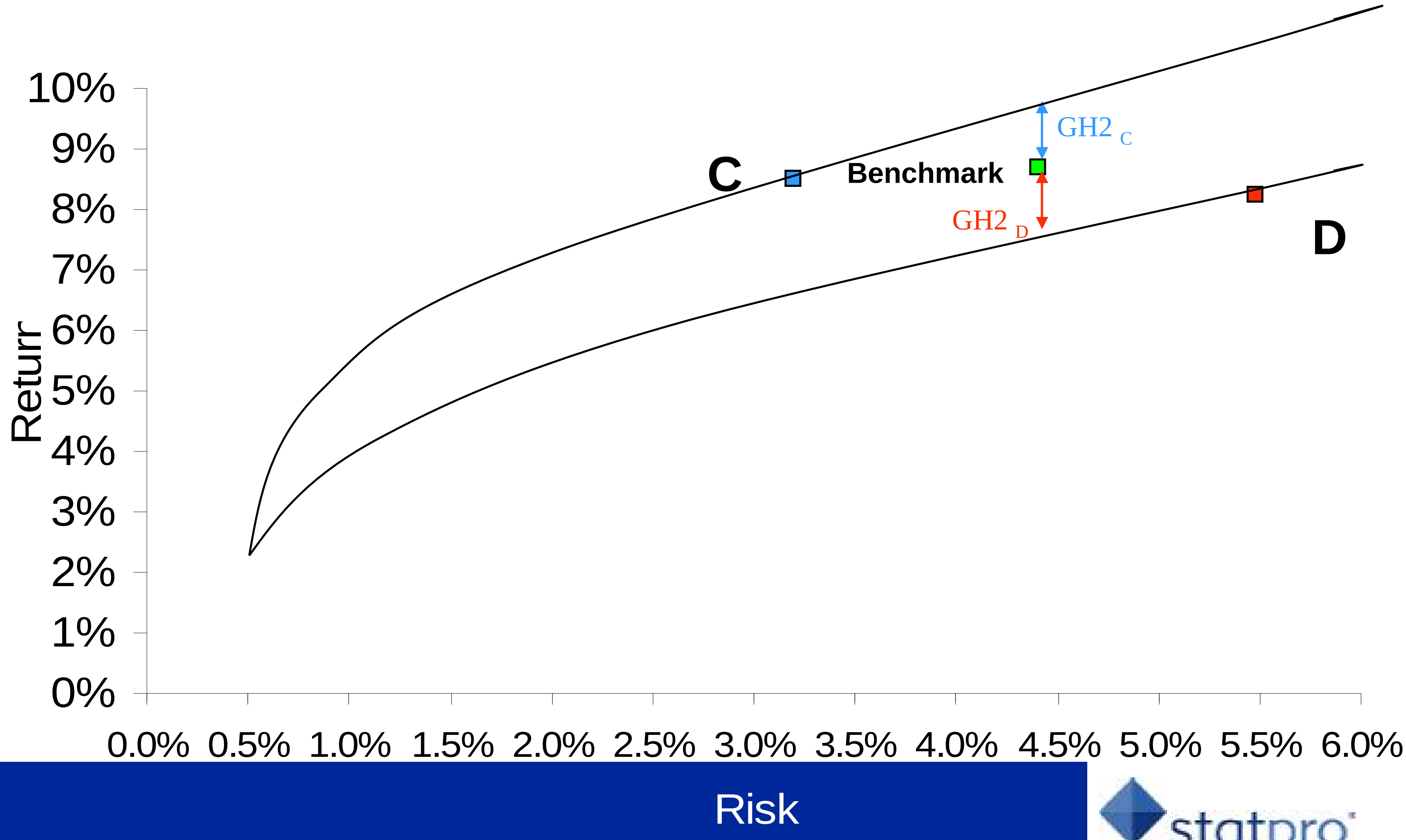
$$d = (\beta^F - \beta_P) \times (r_M - r_F)$$

Net Selectivity (s) $s = \alpha - d$

GH 1 (Graham & Harvey)



GH2 (Graham & Harvey) for M^2



Hedge Funds

- “A hedge fund constitutes an investment program whereby the managers or partners seek absolute returns by exploiting investment opportunities while protecting principal from potential financial loss”

Ineichen (2003)

Risk-adjusted Performance for Hedge Funds

- Investment returns are not normal
  - Particularly hedge funds
- Skewness

$$S = \sum \left(\frac{r_i - \bar{r}}{\sigma_P} \right)^3 \times \frac{1}{n}$$

- Kurtosis

$$K = \sum \left(\frac{r_i - \bar{r}}{\sigma_p} \right)^4 \times \frac{1}{n}$$

- Excess Kurtosis

$$K_E = \sum \left(\frac{r_i - \bar{r}}{\sigma_p} \right)^4 \times \frac{1}{n} - 3$$

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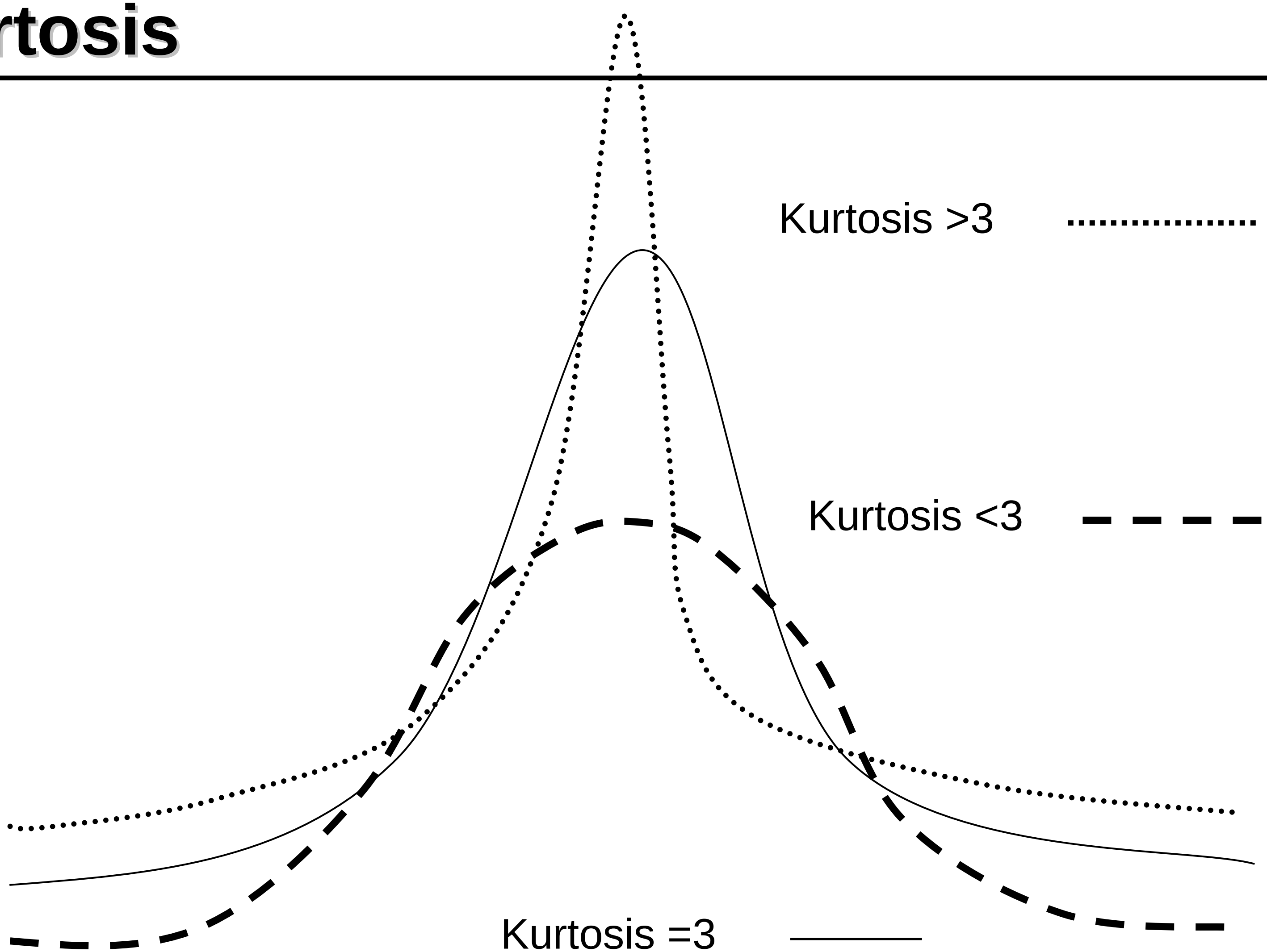
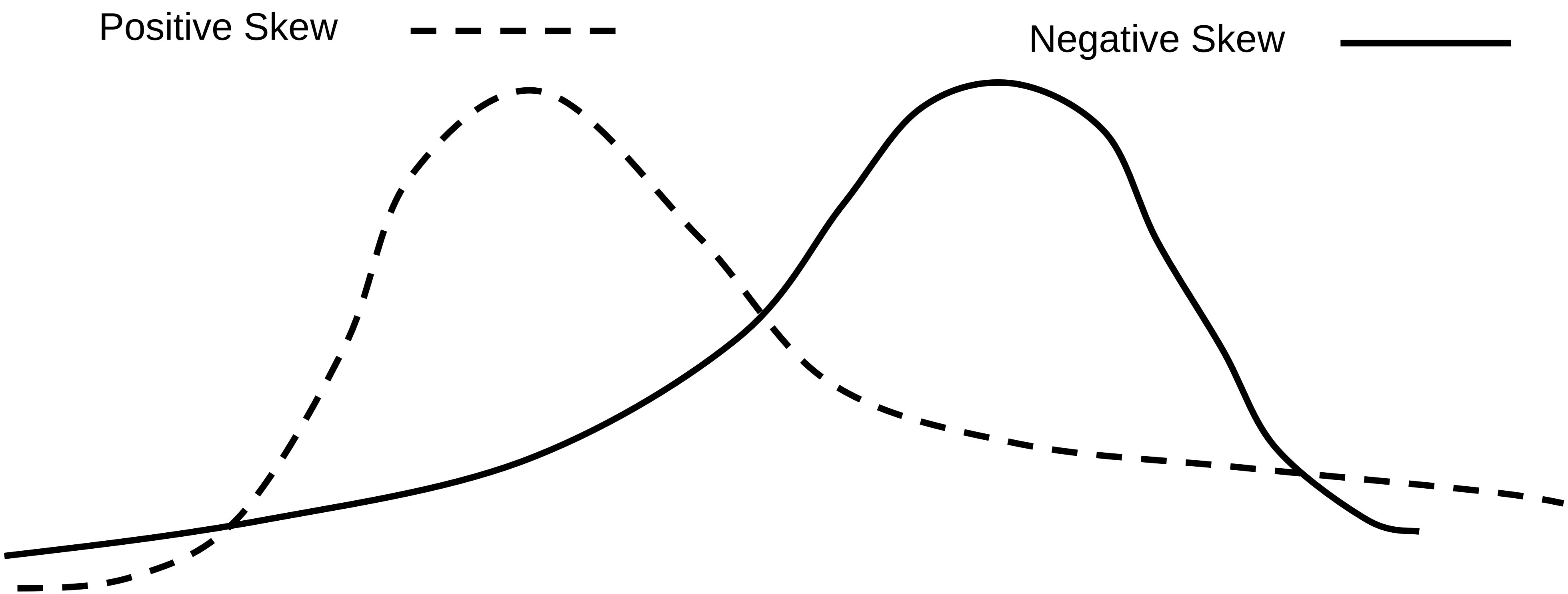
Kurtosis



Skewness



Bera-Jarque Test

- Test for normality

$$BJ = \frac{n}{6} \times \left(S^2 + \frac{K_E^2}{4} \right)$$

- Reject if exceeds 5.99 (95%)
- Reject if exceeds 9.21 (99%)

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Adjusted Sharpe Ratio

$$ASR = SR \times \left[1 + \left(\frac{S}{6} \right) \times SR - \left(\frac{K-3}{24} \right) \times SR^2 \right]$$

Downside Risk

$$\text{Sortino Ratio} = \frac{r_P - r_T}{\sigma_{down}}$$

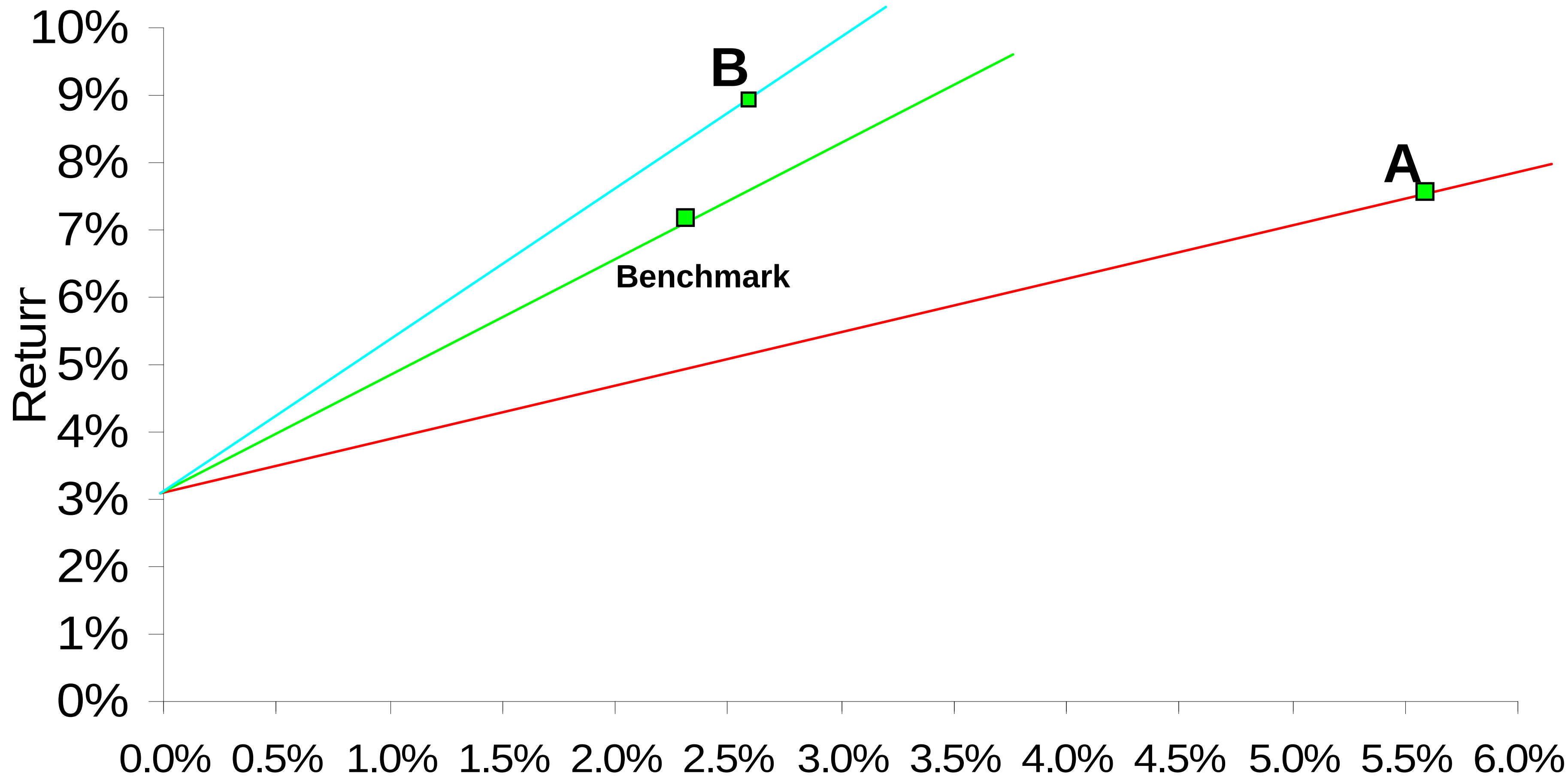
σ_D=Downside semi-standard deviation

r_T =Mimimum acceptable (or Target) return

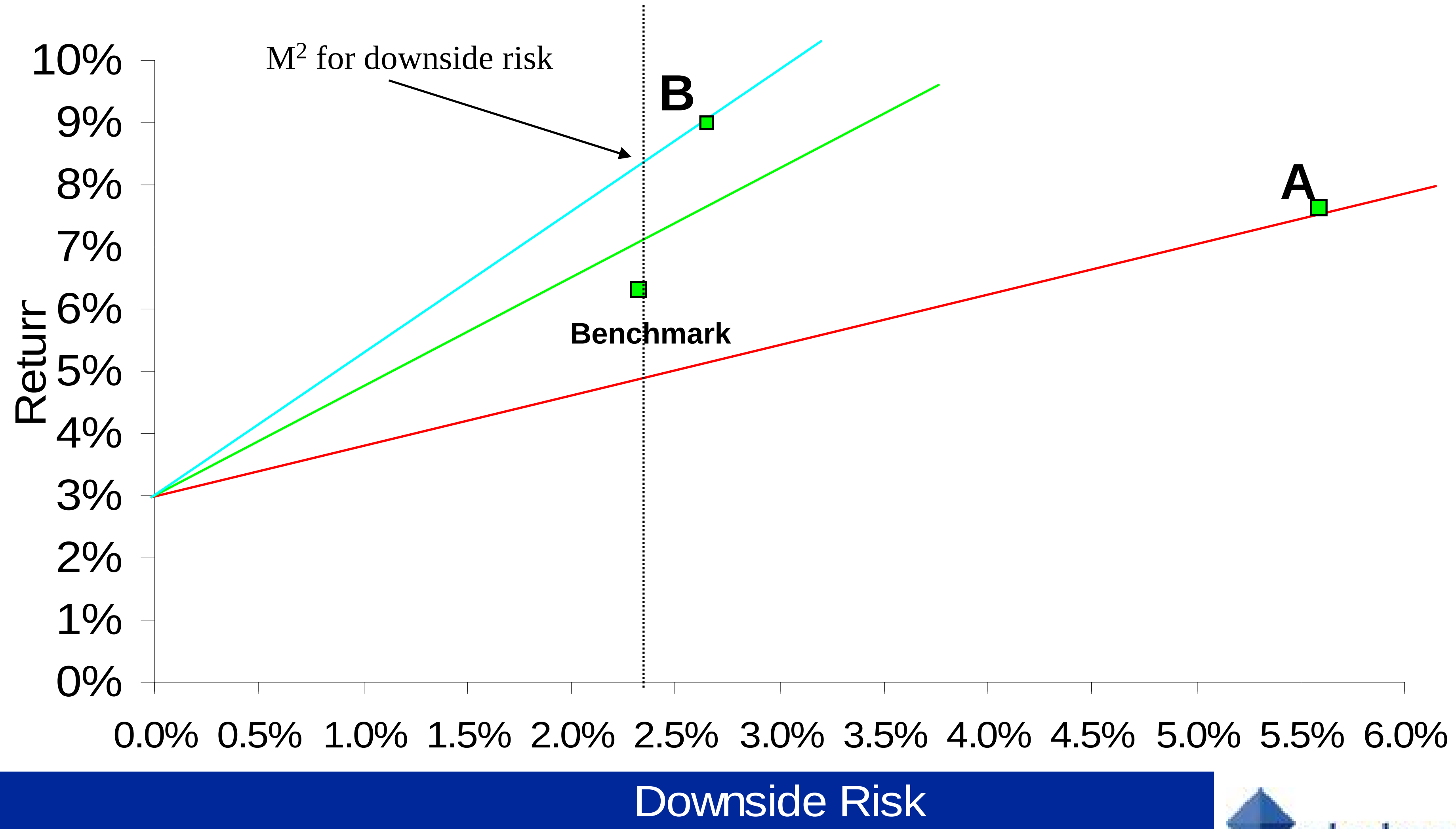
$$\sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min\left[(r_i - r_T),0\right]^2}{n}}$$

- Fewer data points
- Less information
  - Does upside dispersion matter?

Sortino Ratio



M^2 *(for downside risk)*



Upside Potential Ratio (Discrete)

UPR= Upside Potential / Downside Risk

$$UPR = \frac{\sum_{i=1}^{i=n} \max\left(r_i - r_T, 0\right) \Big/ n}{\sigma_D}$$

Omega Ω

$$\Omega = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum \max(r_T - r_i, 0)} = \frac{\text{Upside Potential}}{\text{Downside Potential}}$$

- Gain-Loss Ratio (Bernardo & Ledoit $r_T = 0$)
- Omega-Sharpe Ratio

$$= \frac{r_P - r_T}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} = \Omega - 1$$

Prospect Ratio

$$= \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \left(Max\left(r_i, 0\right) + 2.25 \times Min\left(r_i, 0\right) \right) - r_T}{\sigma_D}$$

- Penalises loss greater than gain
- Based on Prospect Theory

Value at Risk (VaR)

- **Maximum loss in cash terms over a finite period given a certain level of confidence (say 95%)**
- Typically ex-ante
- Volatility is an input - not an output
- Similar to ex-ante tracking error

VaR

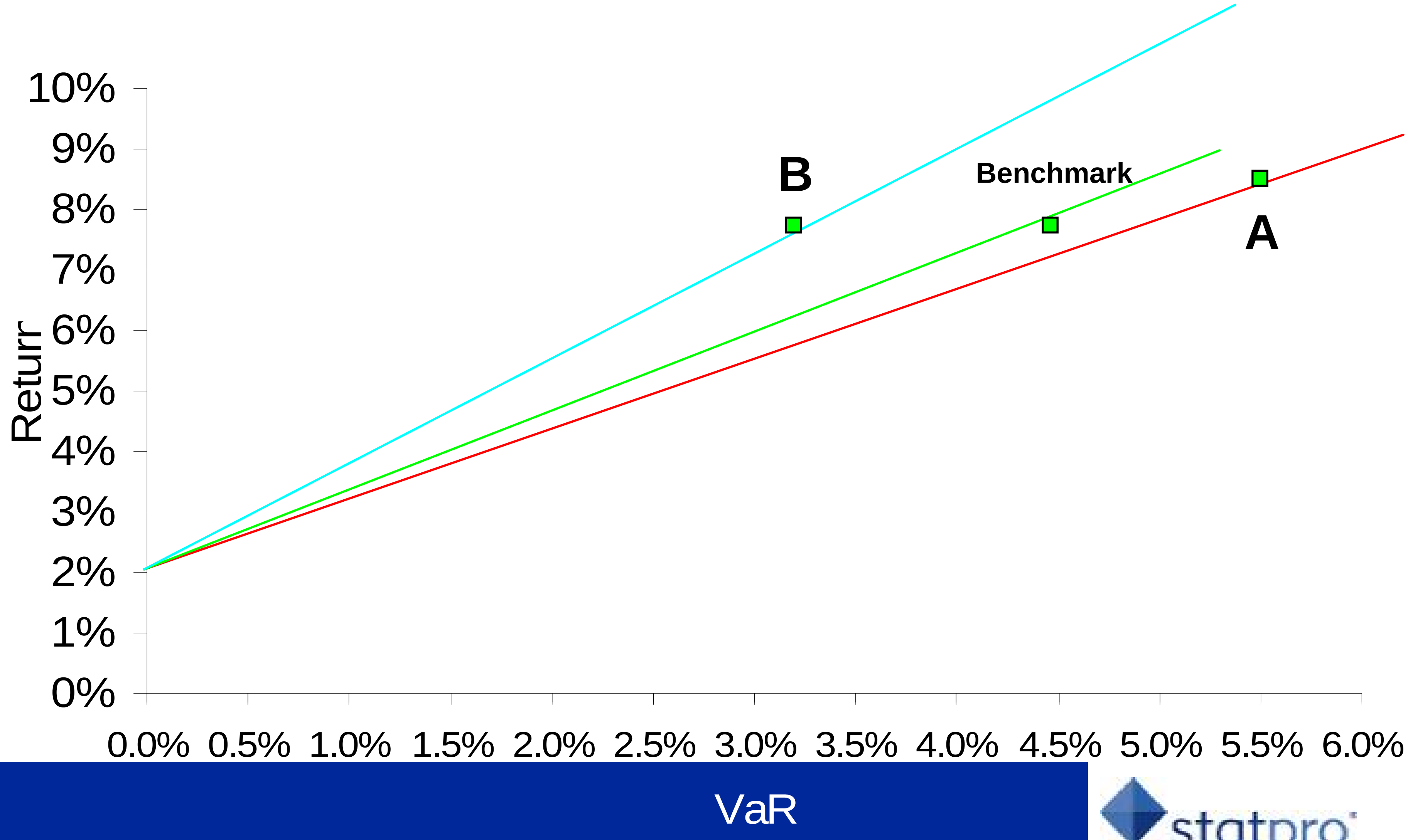
- Historical Simulation
  - Re-organises actual historical returns
- Monte Carlo Simulation
  - Multiple hypothetical trials
- Variance- Co-variance
  - Assumes normality

Return on VaR

Return on VaR

$$R_{VaR} = \frac{r_P - r_F}{VaR\ (\%)}$$

Return on VaR



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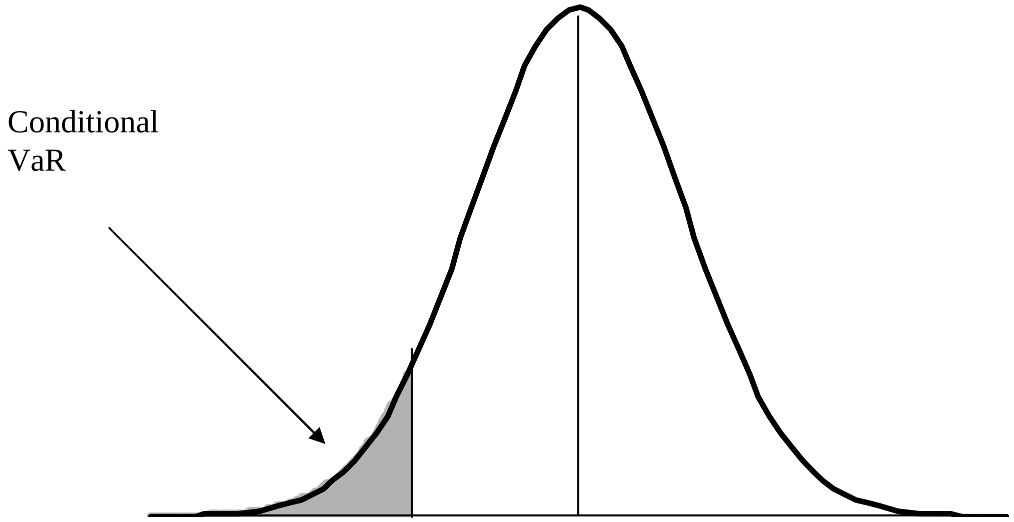
Conditional VaR (*Expected Shortfall*)



Conditional Sharpe Ratio

- Conditional Sharpe Ratio $= \dfrac{r_P - r_F}{\mathrm{CVaR}}$

Modified Va*R*

- Va*R* modified for Kurtosis & Skewness
- Cornish-Fisher expansion

$$\mathrm{MVa}R = \bar{r}_p + \left[z_c + \frac{z_c^2 - 1}{6} \times S + \frac{z_c^3 - 3z_c}{24} \times K_E - \frac{2z_c^3 - 5z_c}{36} \times S^2 \right] \times \sigma$$

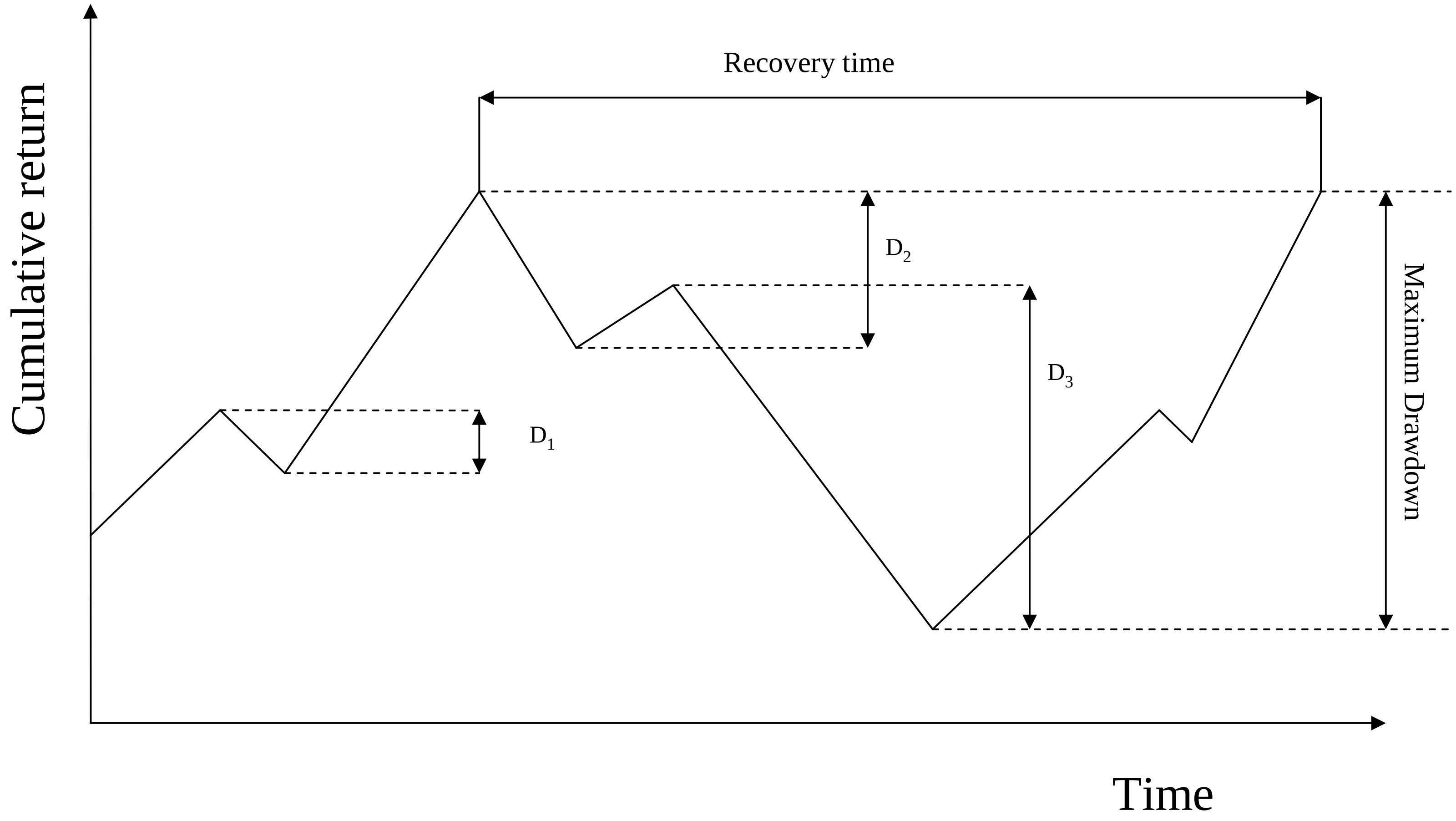
$z_c =$ -1.96 with 95% confidence

$z_c =$ -2.33 with 99% confidence

Modified Sharpe Ratio

- Modified Sharpe Ratio $= \dfrac{r_P - r_F}{\text{MVaR}}$

Drawdown



Recovery time
Cumulative return
D2
D3
D1
Maximum Drawdown
Time

Sterling, Calmar & Burke Ratios

Sterling Ratio $= \dfrac{r - r_F}{\bar{r}_D}$

Calmar Ratio $= \dfrac{r - r_F}{r_D^{Max}}$

Burke ratio $= \dfrac{r - r_F}{\sqrt{\sum r_{Dj}^2}}$

$\bar{R}_D =$ Average Drawdown

$R_D^{Max} =$ Maximum Drawdown

$R_{Dj} =$ Drawdown j

Drawdown = losing period

Versions of the Sterling Ratio

- Multiple Variations
- Original (Deanne Sterling Jones)

$$\frac{r_p}{\bar{r}_D + 10\%}$$

- Average or Largest Individual Drawdown (say 3 or 5)
- Average annual Maximum Drawdown

Sterling-Calmar Ratio

Sterling-Calmar Ratio = $\dfrac{r_p - r_F}{\bar{r}_D^{Max}}$

Combines concepts in Calmar & Sterling Ratios

Pain & Ulcer Indexes

- Reflect pain & worry since high water mark
- Pain Index $$PI = \sum_{i=1}^{i=n} \frac{|D'_i|}{n}$$
- Ulcer Index $$UI = \sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}$$

D'_i = drawdown since pervious peak in period i

Pain & Martin Ratios

- Sharpe type ratios

- Pain Ratio

$$PR = \frac{r_P - r_F}{\sum_{i=1}^{i=m} \frac{D'_i}{n}}$$

- Martin Ratio

  (or Ulcer Performance Ratio)

$$MR = \frac{r_P - r_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}}$$

Duration

- Duration is a Fixed Income Risk Measure
  - Macaulay Duration
  - Macaulay-Weil Duration
  - Modified Duration
  - Effective Duration
- It measures price sensitivity to changes in interest rates

$$Duration \quad \beta = \frac{D_P}{D_B}$$

Hurst Index

$$H = \frac{\log(m)}{\log(n)}$$

Where: $$m = \frac{\max(P_i) - \min(P_i)}{\sigma_P}$$

Between 0.5 and 1 - Persistent

Around 0.5 - Totally Random

Between 0 and 0.5 - Anti-Persistent

Risk Adjusted Attribution

$$R - R_F = \alpha + \beta \times \left(R_M - R_F\right)$$

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Risk Adjusted Attribution

	Portfolio Weight	Benchmark Weight	Portfolio Return	Index Return	β
UK	40%	40%	20.0	10.0	1.3
Japan	30%	20%	-5.0	-4.0	1.0
US	30%	40%	6.0	8.0	0.8
Total	100%	100%	8.3	6.4	1.1
Total Excess Return			**=1.79**		

Risk Adjusted Attribution (Asset Allocation)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Index Return	Semi-Notional
UK	40%	40%	20.0	10.0	10.0
Japan	30%	20%	-5.0	-4.0	-4.0
US	30%	40%	6.0	8.0	8.0
Total	100%	100%	8.3	6.4	5.2

Total Excess Return $\frac{1.083}{1.064}$ **-1= 1.79**

Asset (or country) Allocation

UK $$\left[40\% - 40\%\right] \times \left(\frac{1.10}{1.064} - 1 \right) = 0$$

JAPAN $$\left[30\% - 20\%\right] \times \left(\frac{0.96}{1.064} - 1 \right) = -0.97$$

US $$\left[30\% - 40\%\right] \times \left(\frac{1.08}{1.064} - 1 \right) = -0.15$$

TOTAL 0 - 0.97 - 0.15 = -1.13

or alternatively $$\frac{1.052}{1.064} - 1 = -1.13$$

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Systematic Risk

	Portfolio Weight	Benchmark Weight	Portfolio Return	Index Return	Risk-adjusted
UK	40%	40%	20.0	10.0	12.7
Japan	30%	20%	-5.0	-4.0	-4.0
US	30%	40%	6.0	8.0	6.4
Total	100%	100%	8.3	6.4	5.8

Systematic Risk

UK $$[40\%]\times\left(\frac{1.127}{1.10}-1\right)\times\left(\frac{1.10}{1.052}\right)=1.03$$

JAPAN $$[30\%]\times\left(\frac{0.96}{0.96}-1\right)\times\left(\frac{0.96}{1.052}\right)=0.0$$

US $$[30\%]\times\left(\frac{1.0644}{1.08}-1\right)\times\left(\frac{1.08}{1.052}\right)=-0.44$$

TOTAL 1.03 +0.0 - 0.44 = 0.58

or alternatively $$\frac{1.0581}{1.052}-1=0.58$$

Selectivity (or Adjusted Stock Selection)

	Portfolio Weight	Benchmark Weight	Portfolio Return	Index Return	Risk-adjusted
UK	40%	40%	20.0	10.0	12.7
Japan	30%	20%	-5.0	-4.0	-4.0
US	30%	40%	6.0	8.0	6.4
Total	100%	100%	8.3	6.4	5.8

UK $$[40\%]\times\left(\frac{1.20}{1.127}-1\right)\times\left(\frac{1.127}{1.0581}\right)=2.76$$

JAPAN $$[30\%]\times\left(\frac{0.95}{0.96}-1\right)\times\left(\frac{0.96}{1.0581}\right)=-0.28$$

US $$[30\%]\times\left(\frac{1.06}{1.0644}-1\right)\times\left(\frac{1.0644}{1.0581}\right)=-0.12$$

TOTAL 2.76 - 0.28 - 0.12 = 2.35

or alternatively $$\frac{1.083}{1.0581}-1=2.35$$

Concluding Remarks

- **"Risk like beauty is in the eye of the beholder"**
- The basic structure of all composite risk measures is the same**:**

$$\frac{\text{Reward}}{\text{Risk}}$$

- Get it wrong and you're in trouble

Unilever - Merrill

Merrill faces payout to avoid court action

Possible compromise with pension fund could cost £75m

By Simon Targett, Investment Correspondent

Merrill Lynch Investment Managers may have to pay as much as £75m if it wants to stave off an embarrassing High Court action with the £4bn Unilever Superannuation Fund, one of the largest pension schemes in the UK.

The fund – run for employees of Unilever, the Anglo-Dutch group – has brought a £130m lawsuit against the US bank's asset management arm for allegedly mismanaging £1bn of its investments about four years ago.

MLIM, one of the world's biggest fund managers, is understood to be keen to reach a compromise before the case reaches the High Court next week. Bob Doll, MLIM's new president, told the Financial Times last week: "We want [the court case] out of the way, and the sooner the better."

But USF's trustees are preparing to drive a hard bargain. They are understood to have turned down earlier efforts by MLIM to strike a compromise, including an offer of a £20m payment.

"The trustees need to see a big enough number to show that Merrill Lynch have clearly conceded the point that they were in the wrong here," said one person close to Unilever. "The trustees, having got this far and having dared to take on the City establishment, are keen to prove that there was negligence here, and that they were not just sore losers."

USF is unhappy that MLIM's fund managers underperformed an agreed benchmark index by 10.5 per cent in 1997, even though a contract specified that they should not fall more than 3 per cent below the index.

MLIM says this downside threshold was no more than a target and it was not a guarantee. But USF insists it was the product of serious negotiation towards the end of 1996, and MLIM entered into the agreement in the full knowledge of its importance as a "tram line" within which it could invest money in the stock market.

USF – advised by Frank Russell, the US-based pension consultancy – had originally proposed a downside threshold of 2 per cent. It changed this after protests from MLIM, which had, by then, run money for Unilever for nine years.

Talks between Slaughter & May, USF's lawyers, and Simmonds & Simmonds, acting for MLIM, are not expected to be delayed by the collapse of Richard Greenhalgh, chairman of USF's trustees and a senior Unilever executive. Last night, Mr Greenhalgh was still reported to be in hospital, with a suspected heart attack.

www.ft.com/banking

Financial Times

1st October 2001

Appendix - Excel Functions

- Sample excess Kurtosis

$$= \sum \left(\frac{r_i - \bar{r}}{\sigma_p} \right)^4 \times \frac{(n+1) \times n}{(n-1) \times (n-2) \times (n-3)} - 3 \times \frac{(n-1)^2}{(n-2) \times (n-3)}$$

- Sample Skewness

$$= \sum \left(\frac{r_i - \bar{r}}{\sigma_P} \right)^3 \times \frac{n}{(n-1) \times (n-2)}$$