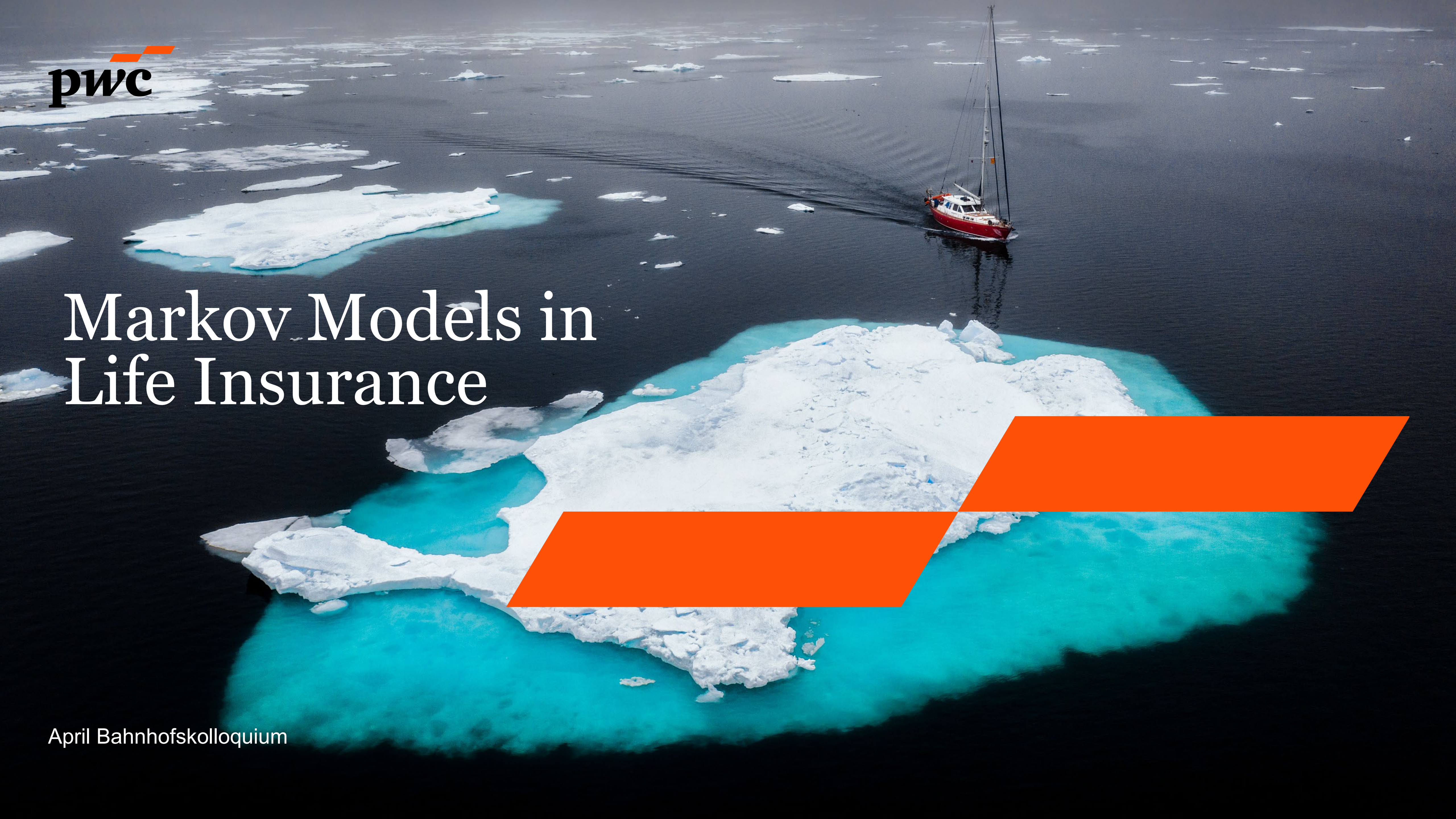
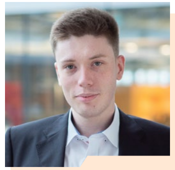


Markov Models in Life Insurance

April Bahnhofskolloquium

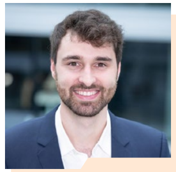
Charting our voyage today

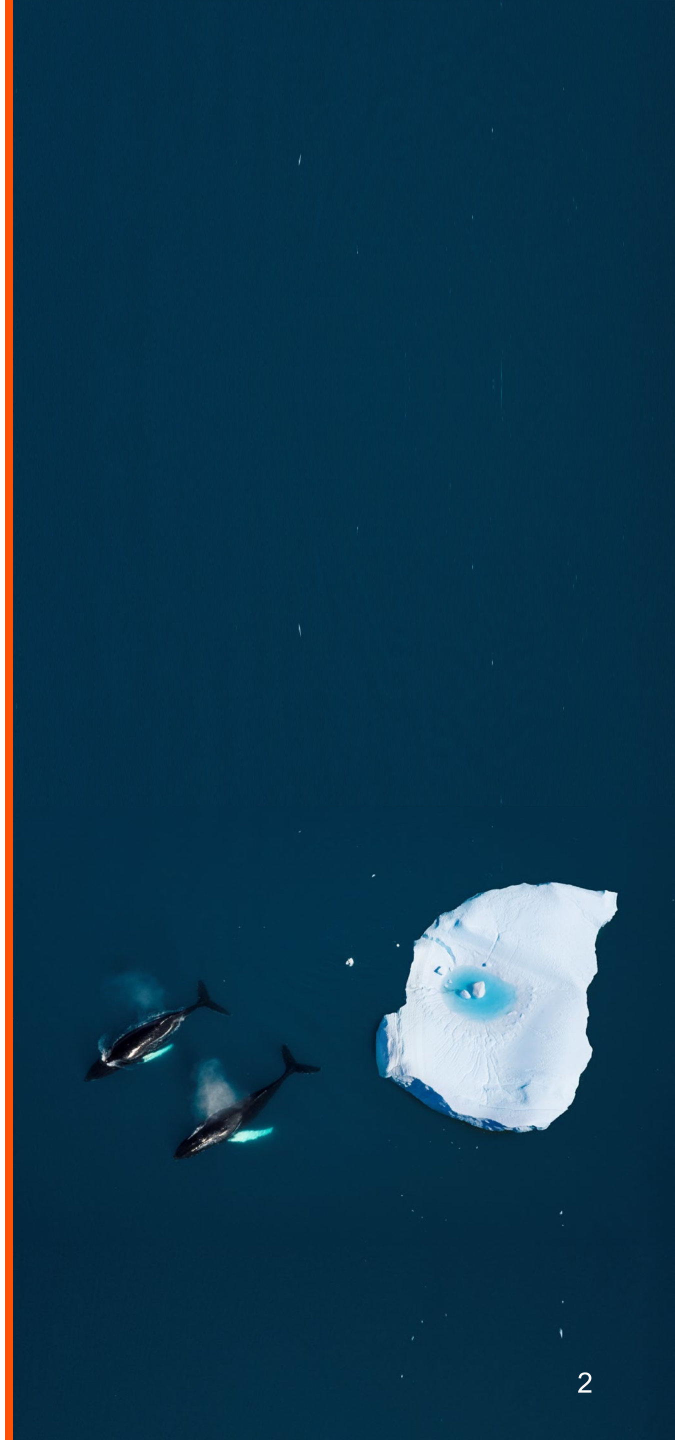
Meet the Crew


Alexandre Allegrezza

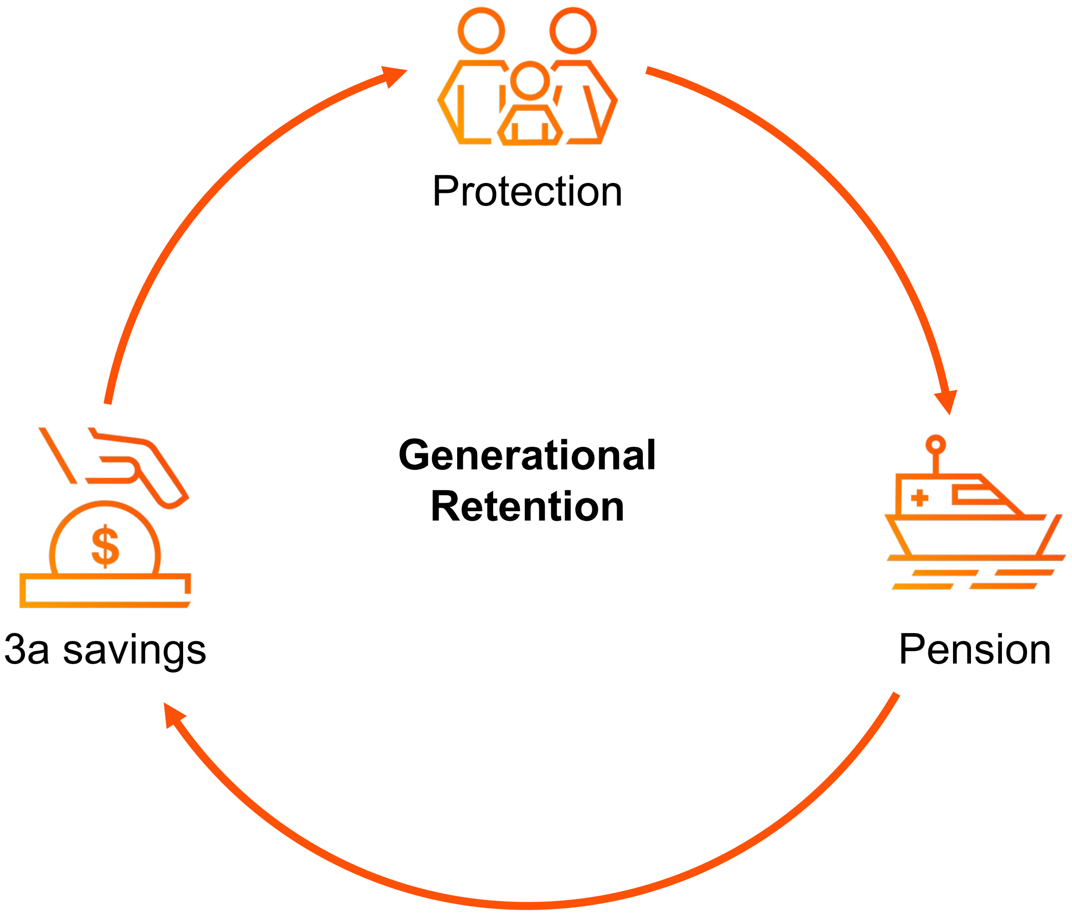

Laura Müllender

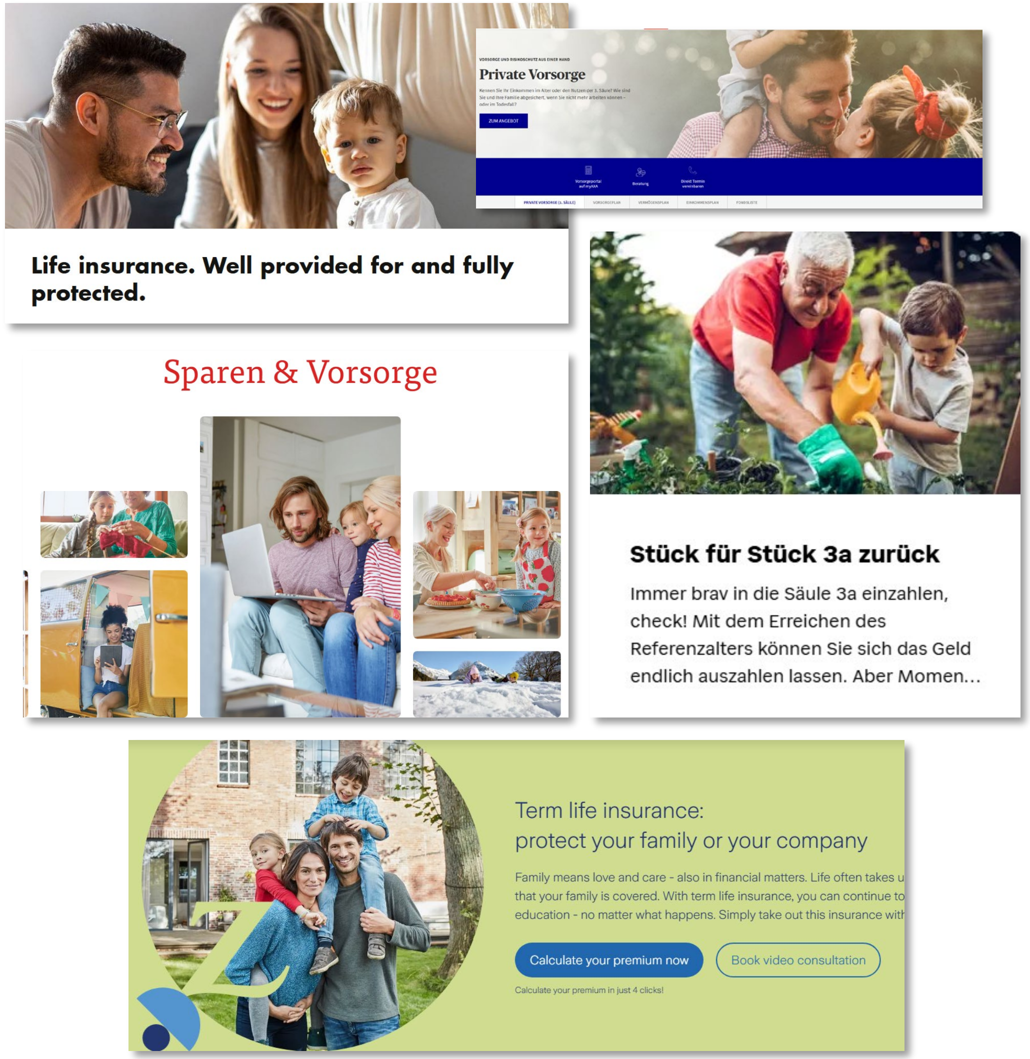

Pheline Sauer


Lucas Hoff

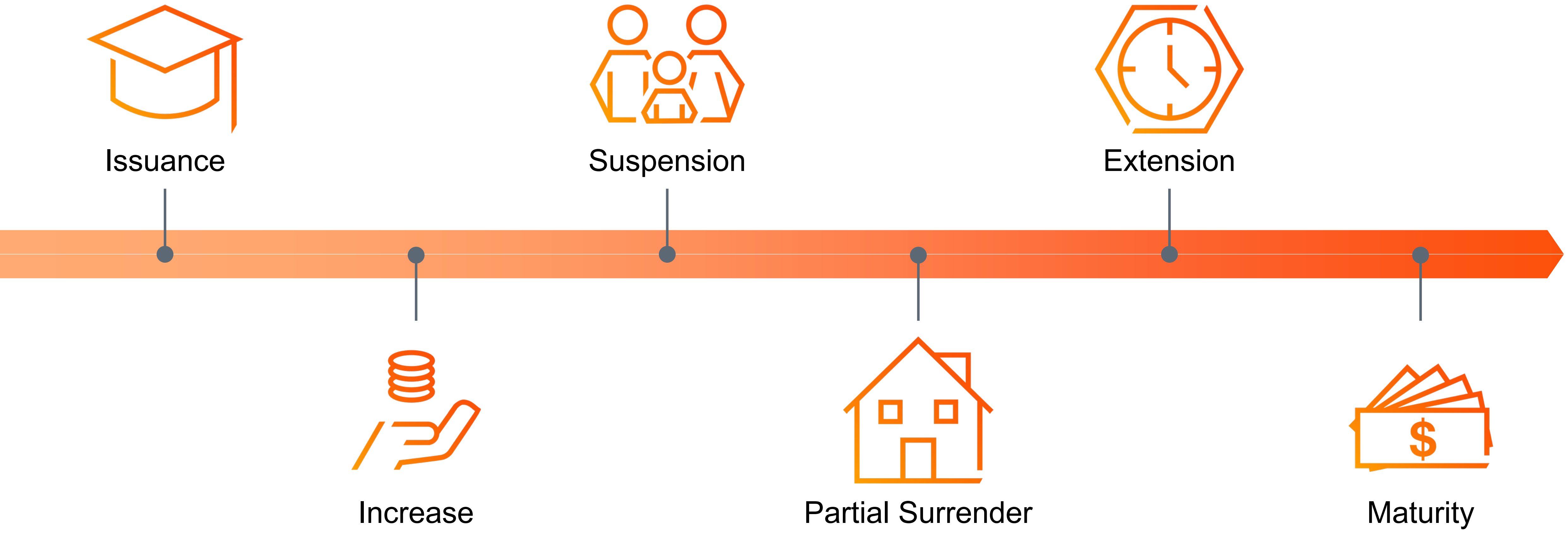


Life insurance is a long-term partnership with the policyholder





So much will happen over a policy lifetime



A typical life insurance policy manages multiple layers of Tariffs

Insurance Policy

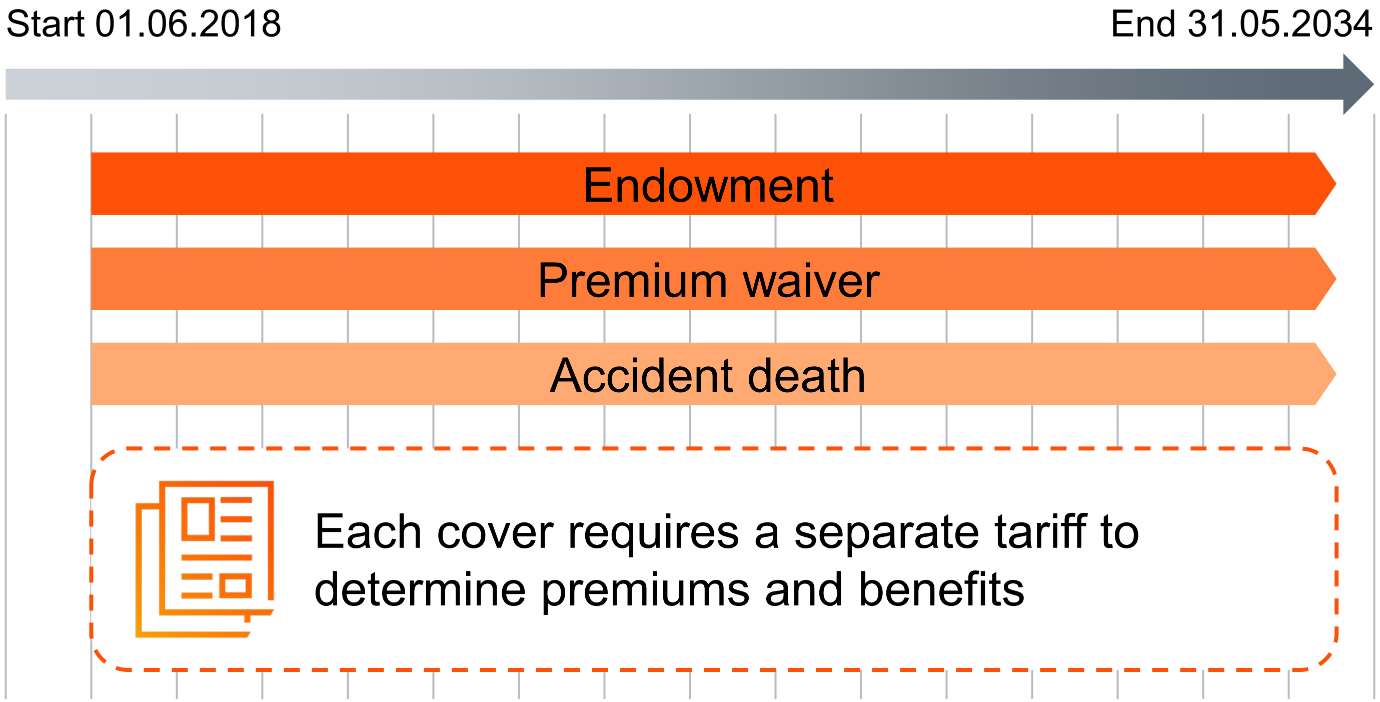
First Name	X
Last Name	XX
Gender	♂
Star Age	49

Policy details:

Start date	01.06.2018
End date	31.05.2034
Main cover	Endowment
Rider 1	Premium waiver
Rider 2	Accident death

Payment	Yearly
Amount	CHF 5’000

Overview of covers



Gross policy premiums

Premium Schedule	CHF 5’000	CHF 5’000	CHF 5’000	CHF 5’000	CHF 5’000	…	CHF 5’000

Anatomy of a life tariff built on commutation numbers

Simplified Endowment Tariff Card

Technical Basis

- Biometric Assumptions (mortality table, etc…)
- Economic Assumptions (technical interest rate, etc…)

Formula Arsenal (10-50 formulas)

Net Premium

$$P^{net}_{x:\overline{n}|} = \frac{SA \cdot A_{x:\overline{n}|}}{\ddot{a}^{(p)}_{x:\overline{m}|}} = SA \cdot \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+m}} \cdot \frac{1}{\alpha(p)}$$

Cost Loading Structure

$$\alpha_1 \cdot SA$$

$$\alpha_2 \cdot P^{gross} \cdot \ddot{a}^{(p)}_{x:\overline{m}|}$$

$$\beta_1 \cdot P^{gross}$$

$$\beta_2 \cdot t \cdot P^{gross}$$

$$\gamma \cdot SA \cdot \ddot{a}_{x:\overline{n}|}$$

$$\gamma_{fix} \cdot \ddot{a}_{x:\overline{n}|}$$

Gross Premium

$$P^{gross} = \frac{SA \cdot A_{x:\overline{n}|} + \alpha_1 \cdot SA + \gamma \cdot SA \cdot \ddot{a}_{x:\overline{n}|} + \gamma_{fix} \cdot \ddot{a}_{x:\overline{n}|}}{(1 - \alpha_2 - \beta_1) \cdot \ddot{a}^{(p)}_{x:\overline{m}|}}$$

Premium Decomposition

$$P^{risk} = SA \cdot \frac{M_x - M_{x+n}}{N_x - N_{x+m}}$$

$$P^{sav} = SA \cdot \frac{D_{x+n}}{N_x - N_{x+m}}$$

$$P^{\alpha_1} = \frac{\alpha_1 \cdot SA}{\ddot{a}^{(p)}_{x:\overline{m}|}}$$

$$P^{\gamma} = \gamma \cdot SA \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}^{(p)}_{x:\overline{m}|}}$$

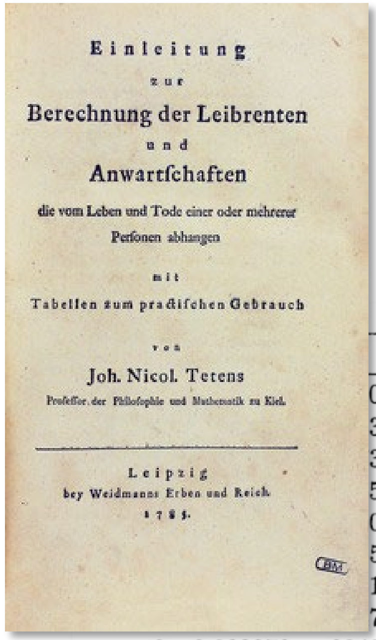
$$P^{\gamma_{fix}} = \gamma_{fix} \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}^{(p)}_{x:\overline{m}|}}$$

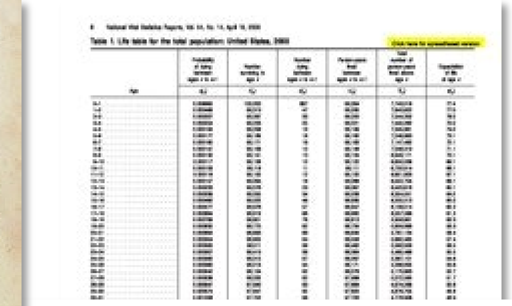
Prospective Reserve and Surrender Value

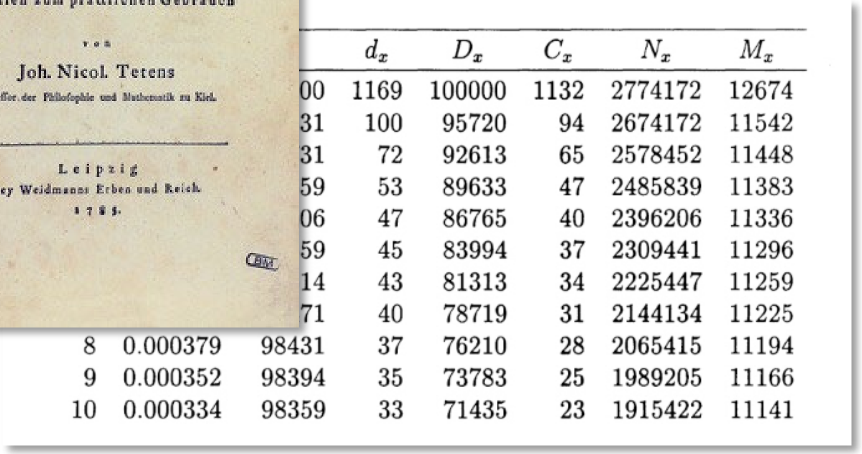
$${}_tV^{net} = SA \cdot A_{x+t:\overline{n-t}|} - P^{net} \cdot \ddot{a}^{(p)}_{x+t:\overline{m-t}|}$$

$$SV_t = \max(0,\ {}_tV^{Zill} - \kappa_t \cdot SA)$$

- Focus on Policy issuance
- Needs partial FINMA approval
- New generations are regularly issued with updated assumptions and formulas

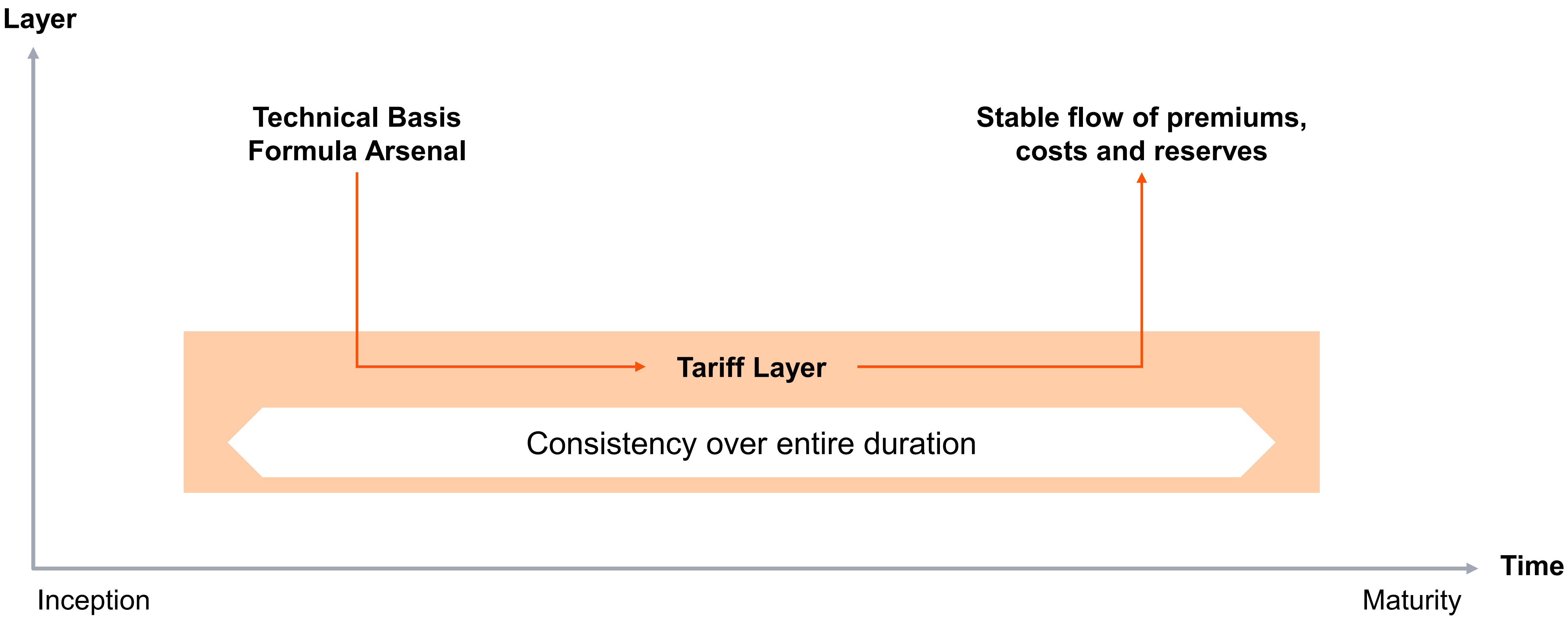




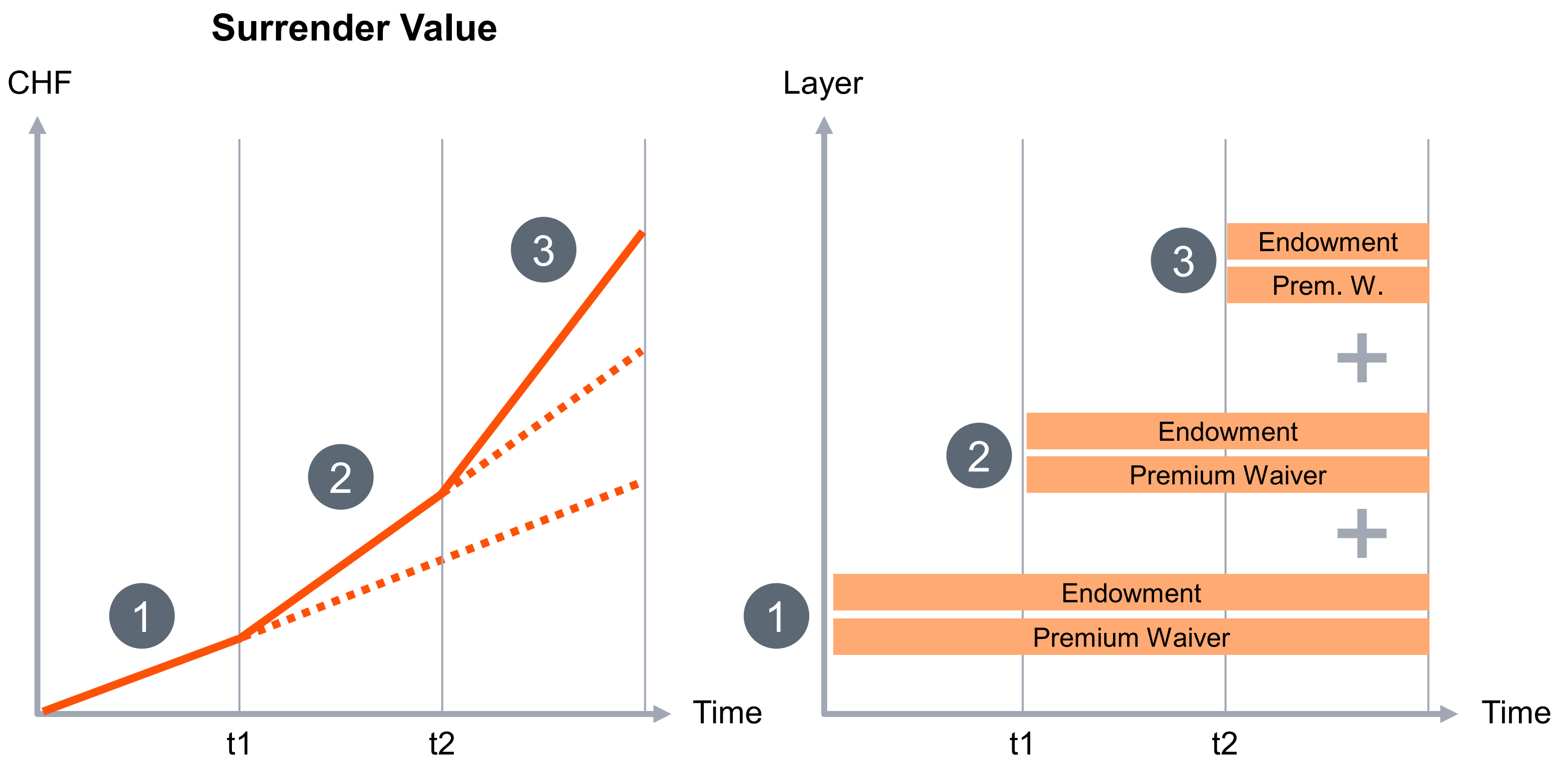


			d_x	D_x	C_x	N_x	M_x
		00	1169	100000	1132	2774172	12674
		31	100	95720	94	2674172	11542
		31	72	92613	65	2578452	11448
		59	53	89633	47	2485839	11383
		06	47	86765	40	2396206	11336
		59	45	83994	37	2309441	11296
		14	43	81313	34	2225447	11259
		71	40	78719	31	2144134	11225
8	0.000379	98431	37	76210	28	2065415	11194
9	0.000352	98394	35	73783	25	1989205	11166
10	0.000334	98359	33	71435	23	1915422	11141

Each individual tariff can be represented as an independent layer



Let’s increase the premium on our policy



1 Policy issuance at a given gross premium or benefit

2 Process to introduce new tariff blocks for first increase

3 Process to introduce new tariff blocks for second increase

Typical Use cases

1. Automatic increase, for example
   a. Inflation linked
   b. 3a maximum cap
2. Permitted increase within same policy

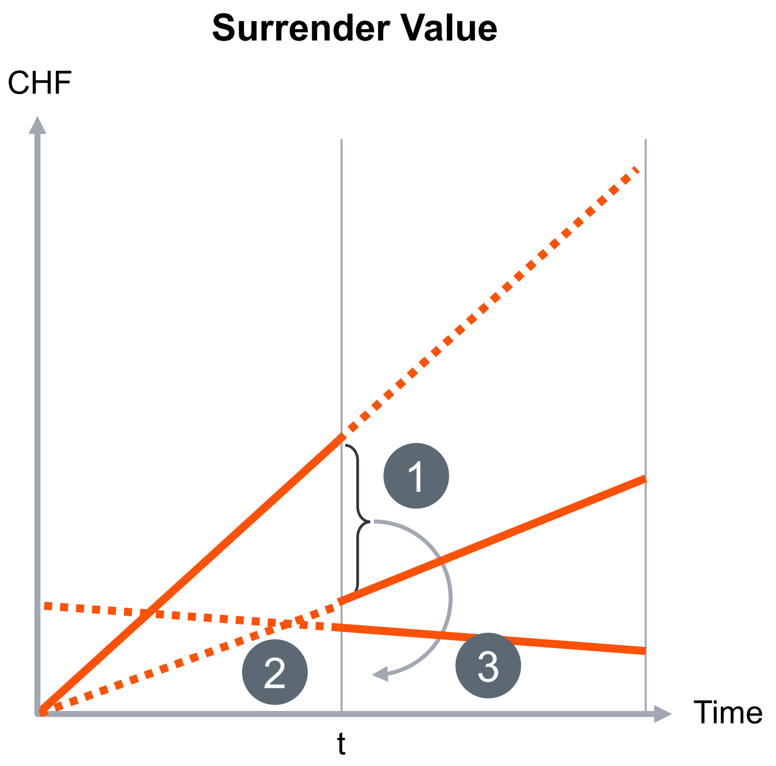
\+ **High Transparency**
Each actuarial value can be traced with high precision in the system.

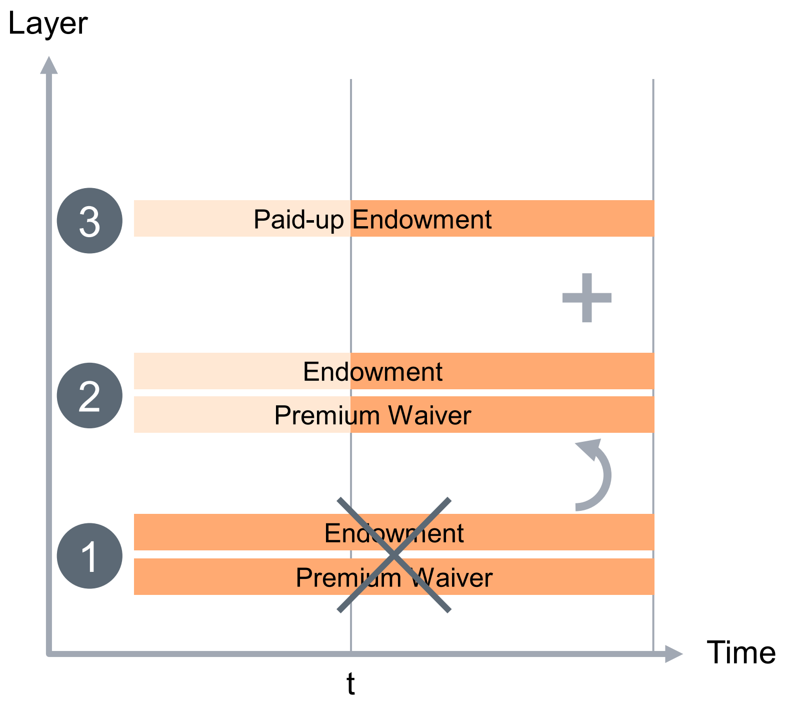
\+ **Easy match for sales commissions**
Allow multi agent/brokers per policy.

− **Difficulty with rebate structures**
Cumbersome to reward policyholders that increase premiums.

− **Handling of spontaneous premiums**
Generate additional layers with separate tariffication. Unclear how to reward policyholders.

Let’s reduce the premium on our policy (Partial Conversion)





1. Determine proportion of reduction to be performed on existing tariff layer to reach target premium or benefit.
2. Process to recalculate new tariff blocks and reperform all actuarial calculations.
3. Process to add new block to model the amount parked from the simulated partial surrender. Recalculate all actuarial fields on converted layer.

Typical Use cases

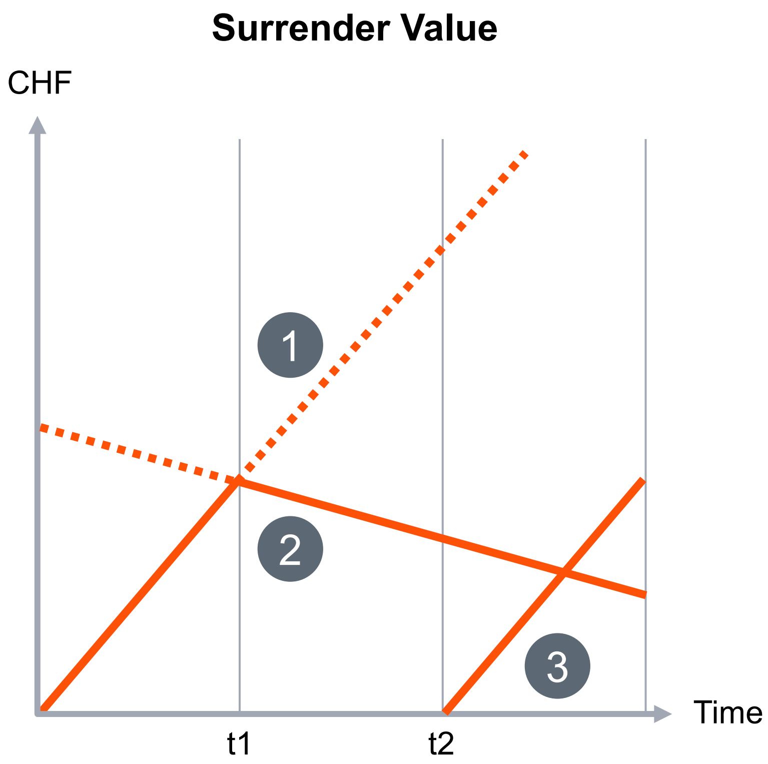
1. Policyholder exerts legal right to reduce regular premium payments at any given time in the year
2. Policyholder protection needs have decreased

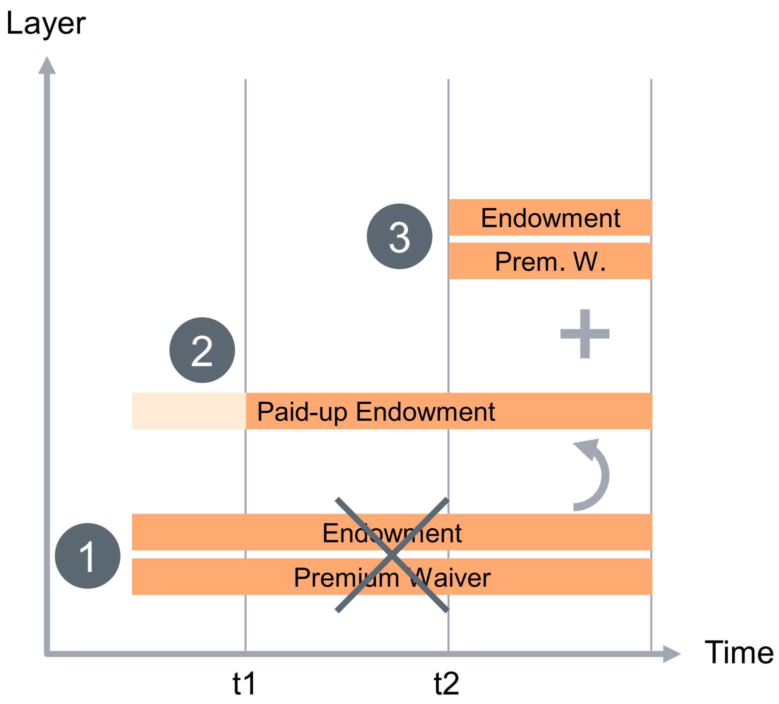
Complexity
Requires multiple subsequent processes exacerbated if the policy already had multiple operations.

Detriment to the policyholder
Leads to forceful amortization of acquisition costs for lack of alternative.

Database pollution
Transparency becomes limited for policies with frequent operations.

Let’s suspend the premium on our policy





1. Process to capture suspension and prematurely end the existing tariff layers.
2. Process to open a conversion layer to park the accumulated surrender value
3. Process to open additional regular premium Tariffs layers that reach the initial premium amount. Unclear if a new underwriting process is required and what tariff generation should be used.

Typical Use cases

1. Retain customer by avoiding a lapse or a conversion of the policy
2. Policyholder has temporary financial difficulties

– **Complexity**
Requires multiple subsequent processes exacerbated if the policy already had multiple operations

– **Detriment to the policyholder**
Lapse penalties are applied even though acquisition costs are levied once more in step 3.

– **High Maintenance**
Existing system business rules might need to be overridden, leading usually to a manual process.

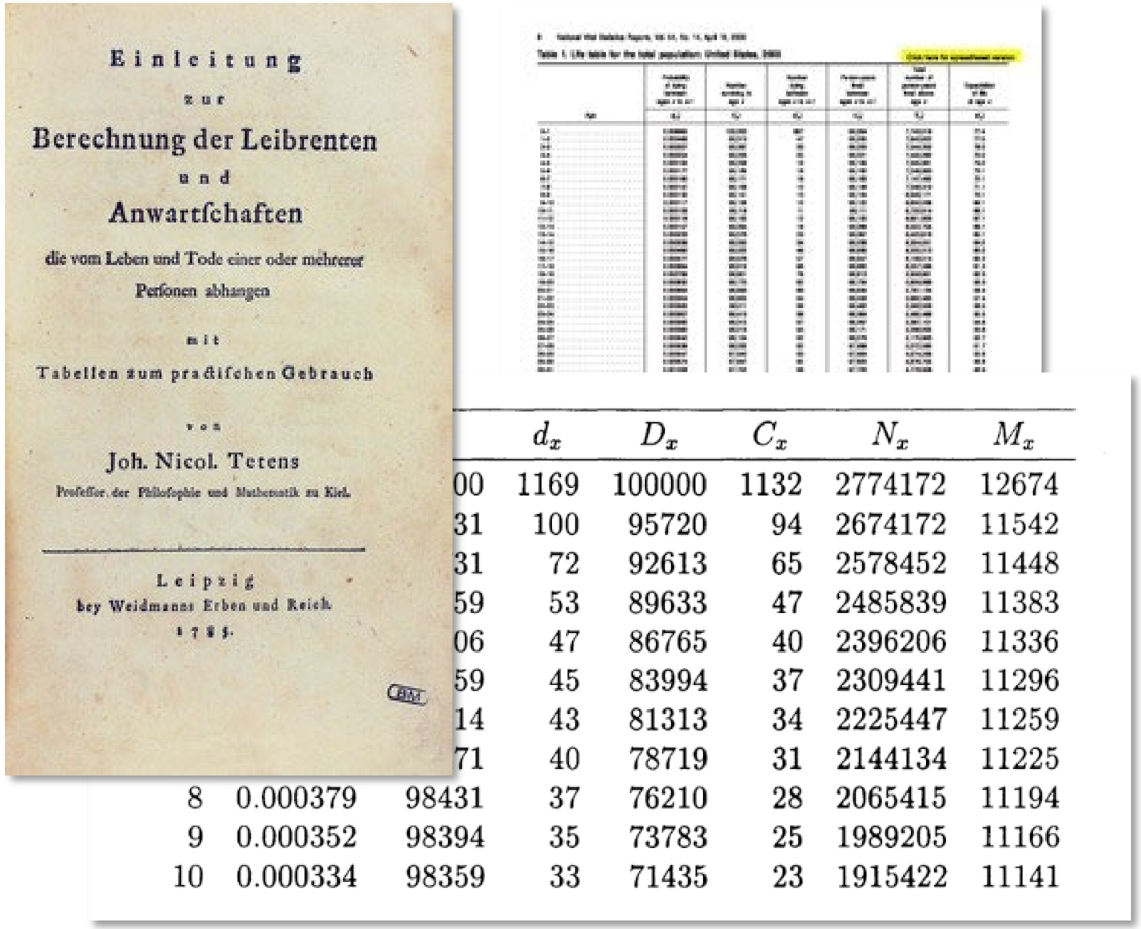
Commutation numbers come with a set of limitations

Advantages from calculating with commutation numbers

- Simple products can be modelled easily
- The meaning of commutation numbers is easy to interpret
- Can be hard coded and uploaded to IT-systems

But there are some limits

- The number of formulas to create grows quickly with the number of products and product complexity
- Higher maintenance efforts to manage the formulas
- Reduced flexibility to introduce new products and manage post issuance operations
- Each policy operation multiplies the number of tariff layers to be maintained



			d_x	D_x	C_x	N_x	M_x
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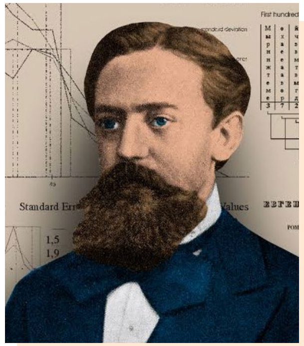
Actuarial Science was a century ahead of the developments in Information Technology

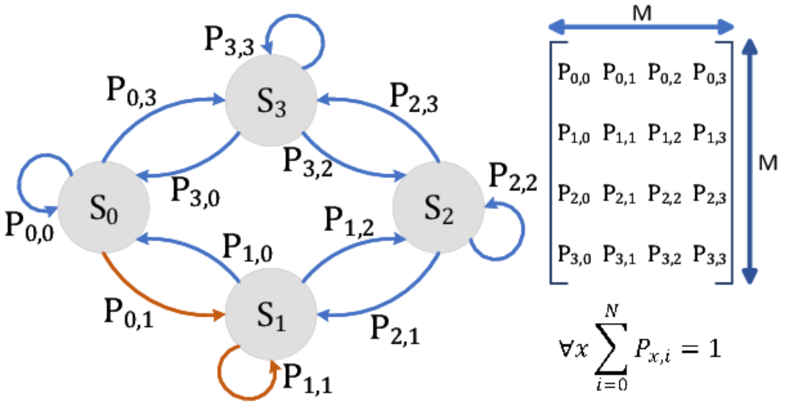


Andrei Andrejewitsch Markow
1856 - 1922

Thorvald Nicolai Thiele
1838 - 1910

Key Points in Brief

- The model describes the possible transition from one contract state to the next, **regardless of potential contract changes**.
- For each time step, a **transition probability** is defined, based on the assumptions of the relevant tariff.
- Using the Markov chain (and with the help of the Thiele recursion), **commutation-based formulas can be rewritten recursively.**
- The key is the **use of vectors** for benefits, gross premiums, and costs in calculating the actuarial reserve.



A new Standard is being established

Leading Software providers are shifting

A different way of expressing commutation numbers

- First use in the 90s, but still too expensive to represent a cost-effective improvement
- 2010 core systems start to reach sufficient performance

Intuition of the state-model using Markov Chains

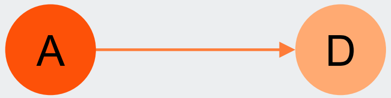
Model Assumptions

Z = State Set {a,b,c,d}
A = Active
D = Dead
I = Disabled
R = Reactivated

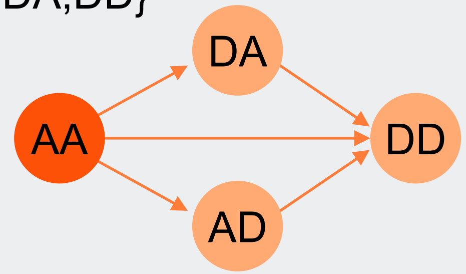
- Model describes the possible Transition from one state to the next
- For each time-step a transition probability is defined

Examples from Life Insurance

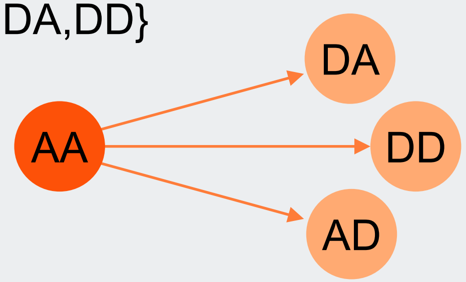
Term Insurance
Z = {A,D}



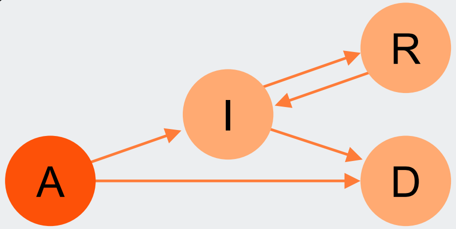
Annuity on 2 Lives
Z = {AA,AD,DA,DD}



Term Insurance on 2 Lives
Z = {AA,AD,DA,DD}



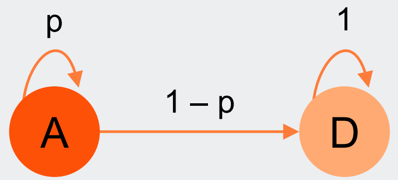
Disability Annuity
Z = {A,I,D,R}



Let’s take the simplest example

Term Life on 1 Life

Z = {A,D}



Probability per time-step:

P(A -> A) = p

P(A -> D) = 1 – p

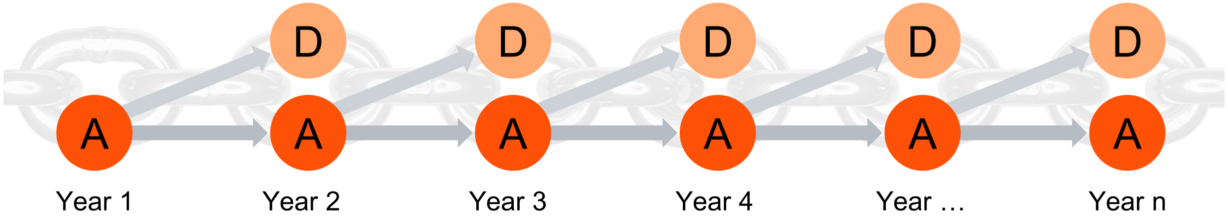
P(D -> D) = 1

Number of Actives and Dead

a(n) = p × a(n-1)

d(n) = (1 – p) × a(n-1) + d(n-1)

When viewed as a process over discrete time steps, the chain becomes apparent



A Insured Person is active D Insured person is deceased

Or expressed in a Matrix-Form:

$$\begin{pmatrix} a(n) \\ d(n) \end{pmatrix} = \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdot \begin{pmatrix} a(n-1) \\ d(n-1) \end{pmatrix} = \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdots\cdot \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdot \begin{pmatrix} a(0) \\ d(0) \end{pmatrix} = \begin{pmatrix} p^n & 0 \\ 1-p^n & 1 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ 0 \end{pmatrix}$$

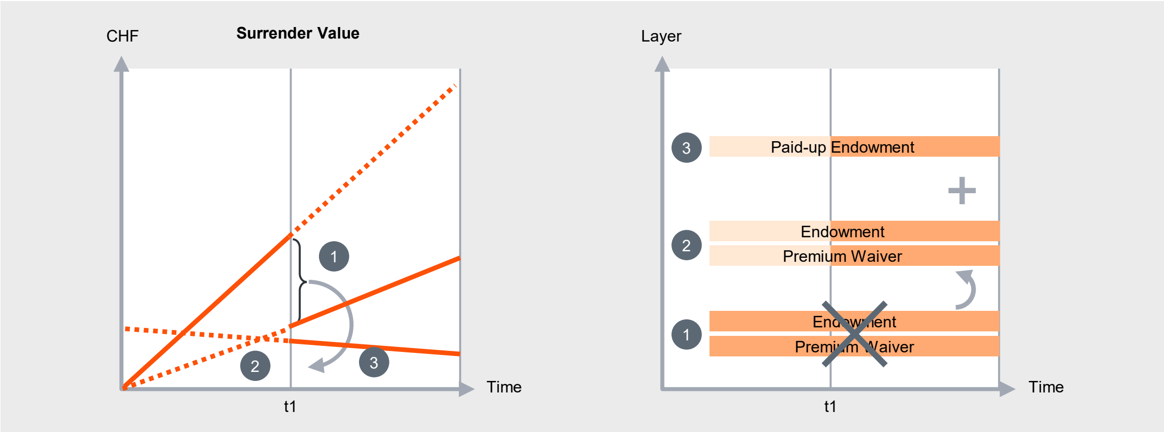
And that’s where the **Thiele Recursion** makes an entrance:

$$V_k(t) = b_k(t) + v(t,t^+) \times \sum_{i \neq k} p_{k,i}(t^+) \left(b_{k,i}(t^+) + V_i(t^+) \right) + v(t,t^+) \times \left(1 - \sum_{i \neq k} p_{k,i}(t^+) \right) \times V_k(t^+)$$

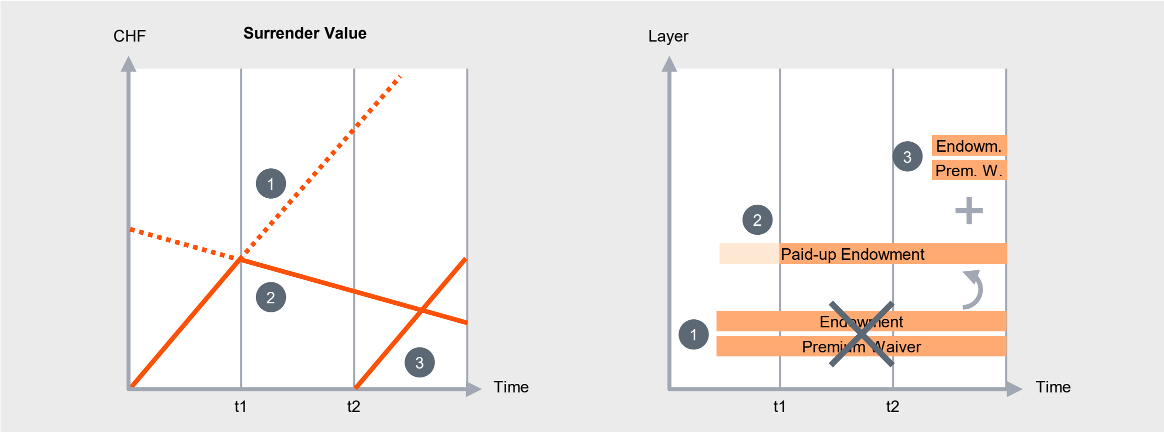
Costs – Gross Premium | Transition probability from state k to state i | Benefit in case of state change | interest from t to t+

Side by side – Suddenly it's all just about updating vector trajectory

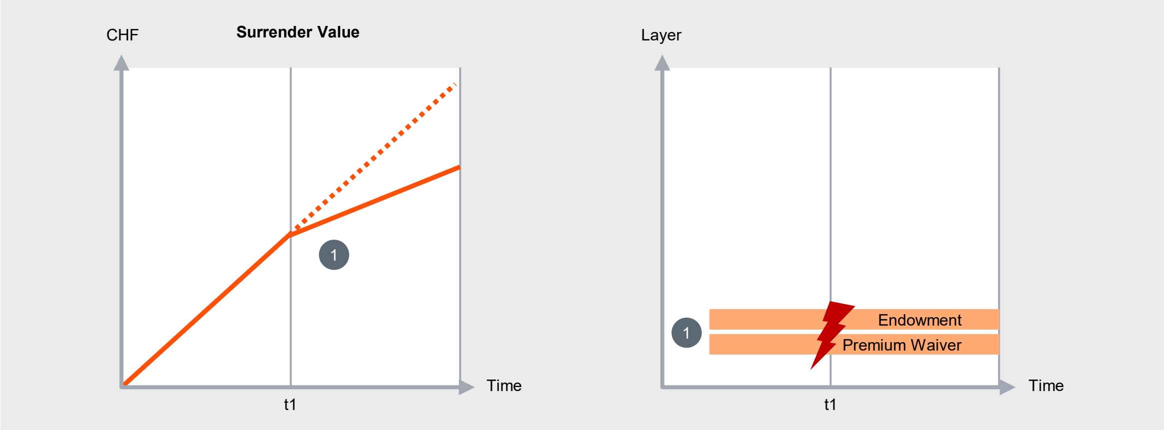
Reduction with commutation numbers



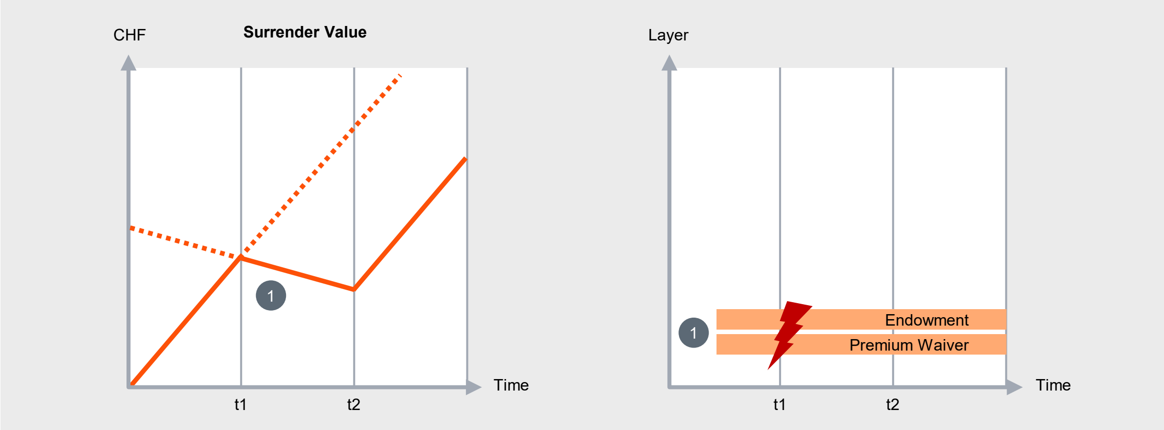
Suspension with commutation numbers

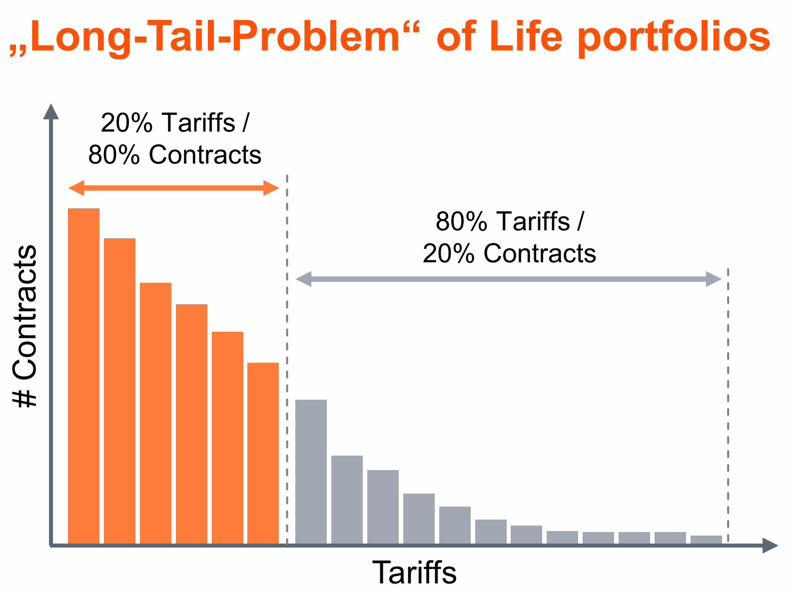


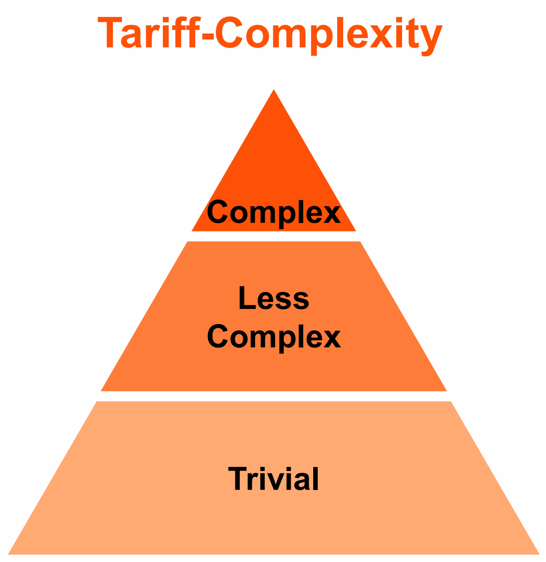
Reduction with Markov approach



Suspension with Markov approach







And now there is a ready toolbox for portfolio rationalization

1

Post extra Reserve – One-time contribution at migration

$$V \pm V^{MIG} = -B \cdot \ddot{a}_{\overline{x+t,n-t}|} + A_{\overline{x+t,n-t}|} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{\overline{x+t,n-t}|} + \gamma_1 \cdot T \cdot \ddot{a}_{\overline{x+t,n-t}|}$$

2

Increase Policyholder Benefit

$$V = -B \cdot \ddot{a}_{\overline{x+t,n-t}|} + A_{\overline{x+t,n-t}|} \pm \hat{A}_{\overline{x+t,n-t}|} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{\overline{x+t,n-t}|} + \gamma_1 \cdot T \cdot \ddot{a}_{\overline{x+t,n-t}|}$$

3

Migration Premium / Migration Discount

$$V = -B \cdot \ddot{a}_{\overline{x+t,n-t}|} \pm MIGBTG \cdot \ddot{a}_{\overline{x+t,n-t}|} + A_{\overline{x+t,n-t}|} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{\overline{x+t,n-t}|} + \gamma_1 \cdot T \cdot \ddot{a}_{\overline{x+t,n-t}|}$$

- Migrate full vectors of benefits , premiums and costs
- No impact on policyholders
- Post migration reserve instead

The Thiele recursion brings most actuarial calculations in one formula

Vector Investment Premium

Alpha & Beta & Gamma & Fix Vectors

Vector Actuarial Reserve ←

$$ {}_tV = vp_{x+t} \times {}_{t+1}V + vq_{x+t} \times {}_tVS + Cost\ Premiums - {}_t\pi $$

→ Vector Gross Premium

Vector Risk Premium

Product Flexibility

New Product Design possibilities as it becomes possible to offer non-linear benefits and premiums, while enabling self service possibilities.

International relevance as the model is neutral to Swiss specific requirements and goes back to scientific fundamentals. One formula for all Tariffs.

Process Streamlining

Need fewer resources to define requirements, maintain and update the system and implement legal changes, due to single formula shared by all operations.

Faster execution of operations, as less manual steps are needed, due to the ability to influence calculations directly via the recursive vectors.

Migration Simplification

Ensure Regulatory Compliance of guaranteed benefits and premiums due to vectorial approach that can also be migrated from legacy.

Faster Migration as we can rationalize and reduce the amount of Tariffs to be migrated, due to the ability of merging individual tariff blocks with the recursion.

The theory looks simple, but what about real examples ?

Demo

> A single Markov chain has the potential of handling the entire life portfolio

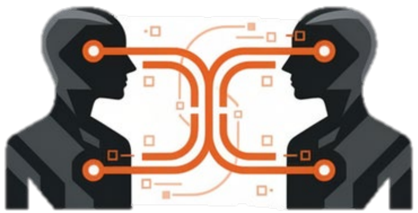
Markov Models in life insurance, the key take-aways

1 Simplification of Processes

2 Cut tariff formula maintenance

3 Game changer for system migration

Next Developments



September 2026
- Full agent to agent Integration PoC
- AWS cloud deployment

November 2026
- Paper Publication in Tokyo
- Full GitHub release