

Markov Models in Life Insurance

Charting our voyage today

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Meet the Crew



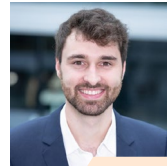
**Alexandre
Allegrezza**



**Laura
Müllender**



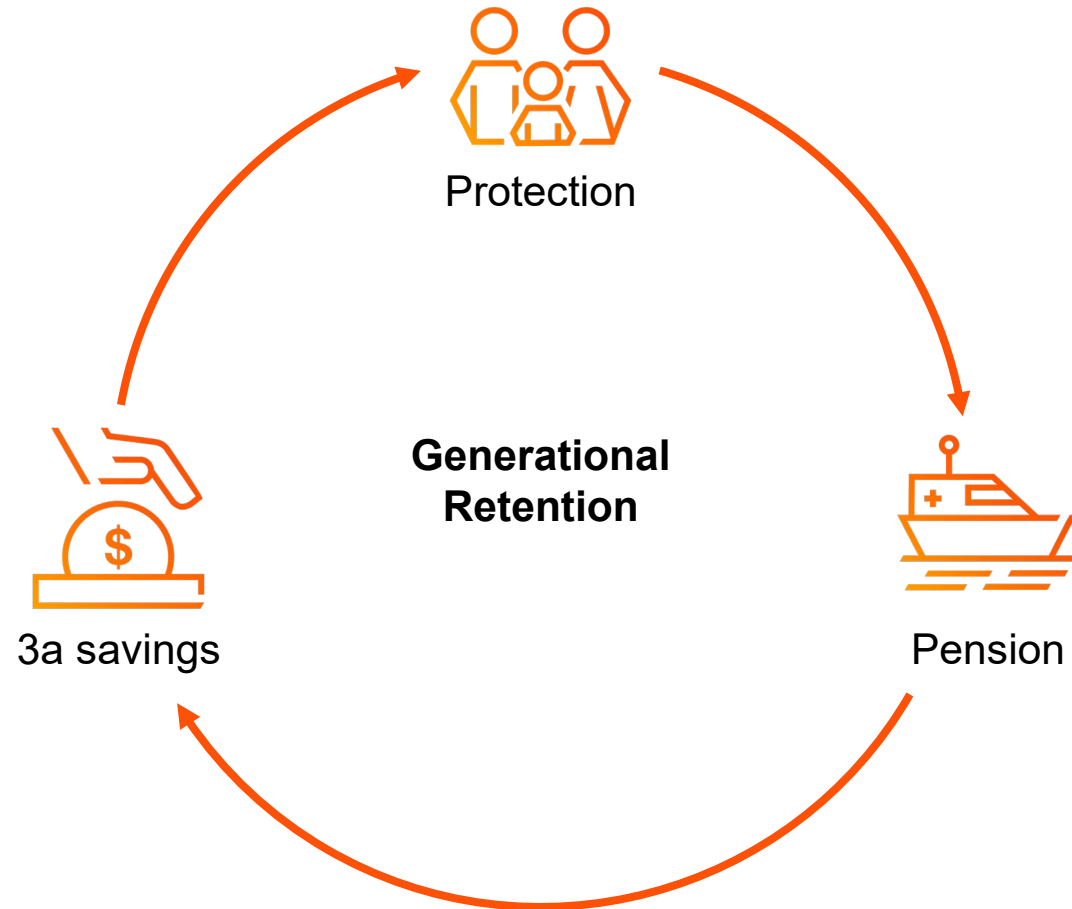
**Pheline
Sauer**



**Lucas
Hoff**



Life insurance is a long-term partnership with the policyholder



Private Vorsorge

Life insurance. Well provided for and fully protected.

Sparen & Vorsorge

Stück für Stück 3a zurück

Immer brav in die Säule 3a einzahlen, check! Mit dem Erreichen des Referenzalters können Sie sich das Geld endlich auszahlen lassen. Aber Momen...

Term life insurance: protect your family or your company

Family means love and care - also in financial matters. Life often takes us that your family is covered. With term life insurance, you can continue to education - no matter what happens. Simply take out this insurance with

Calculate your premium now

Book video consultation

Calculate your premium in just 4 clicks!

So much will happen over a policy lifetime



Issuance



Suspension



Extension



Increase




Partial Surrender



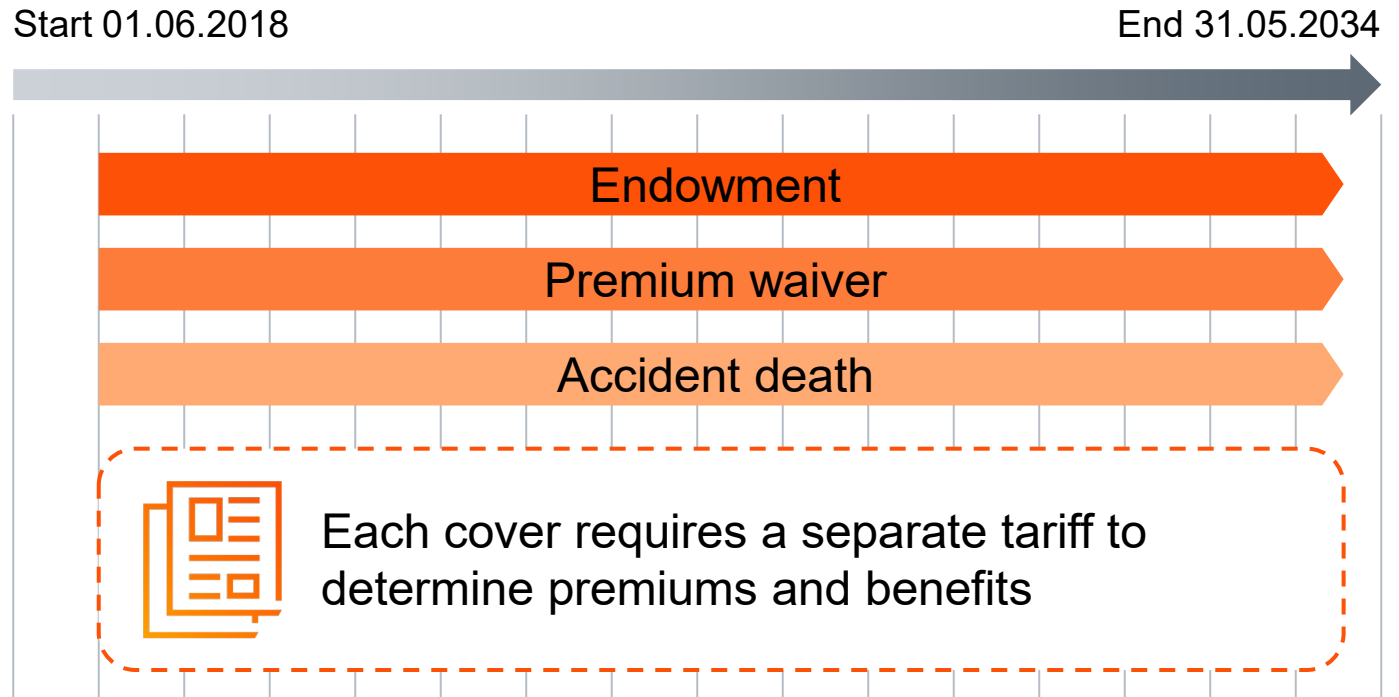
Maturity

A typical life insurance policy manages multiple layers of Tariffs

Insurance Policy

	First Name	X
	Last Name	XX
	Gender	♂
	Star Age	49
Policy details:		
Start date	01.06.2018	
End date	31.05.2034	
Main cover	Endowment	
Rider 1	Premium waiver	
Rider 2	Accident death	
Payment	Yearly	
Amount	CHF 5'000	

Overview of covers



Gross policy premiums

Premium Schedule	CHF 5'000	CHF 5'000	CHF 5'000	CHF 5'000	CHF 5'000	...	CHF 5'000
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Anatomy of a life tariff built on commutation numbers

Simplified Endowment Tariff Card

Technical Basis

- Biometric Assumptions (mortality table, etc...)
- Economic Assumptions (technical interest rate, etc...)

Formula Arsenal (10-50 formulas)

Net Premium

$$P_{x:\overline{n}|}^{net} = \frac{SA \cdot A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(p)}} = SA \cdot \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \cdot \frac{1}{\alpha^{(p)}}$$

Cost Loading Structure

$$\alpha_1 \cdot SA$$

$$\alpha_2 \cdot P^{gross} \cdot \ddot{a}_{x:\overline{n}|}^{(p)}$$

$$\beta_1 \cdot P^{gross}$$

$$\beta_2 \cdot t \cdot P^{gross}$$

$$\gamma \cdot SA \cdot \ddot{a}_{x:\overline{n}|}$$

$$\gamma_{fix} \cdot \ddot{a}_{x:\overline{n}|}$$

Gross Premium

$$P^{gross} = \frac{SA \cdot A_{x:\overline{n}|} + \alpha_1 \cdot SA + \gamma \cdot SA \cdot \ddot{a}_{x:\overline{n}|} + \gamma_{fix} \cdot \ddot{a}_{x:\overline{n}|}}{(1 - \alpha_2 - \beta_1) \cdot \ddot{a}_{x:\overline{n}|}^{(p)}}$$

Premium Decomposition

$$P^{risk} = SA \cdot \frac{M_x - M_{x+n}}{N_x - N_{x+n}}$$

$$P^{sav} = SA \cdot \frac{D_{x+n}}{N_x - N_{x+n}}$$

$$P^{\alpha_1} = \frac{\alpha_1 \cdot SA}{\ddot{a}_{x:\overline{n}|}^{(p)}}$$

$$P^{\gamma} = \gamma \cdot SA \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(p)}}$$

$$P^{\gamma_{fix}} = \gamma_{fix} \cdot \frac{\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(p)}}$$

Prospective Reserve and Surrender Value

$${}_tV^{net} = SA \cdot A_{x+t:\overline{n-t}|} - P^{net} \cdot \ddot{a}_{x+t:\overline{n-t}|}^{(p)}$$

$$SV_t = \max(0, {}_tV^{Zill} - \kappa_t \cdot SA)$$



- Focus on Policy issuance
- Needs partial FINMA approval
- New generations are regularly issued with updated assumptions and formulas

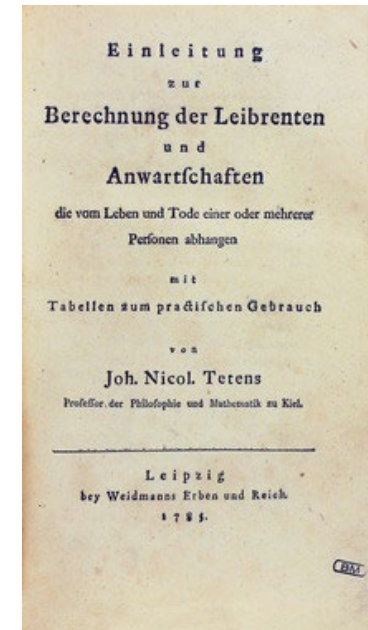
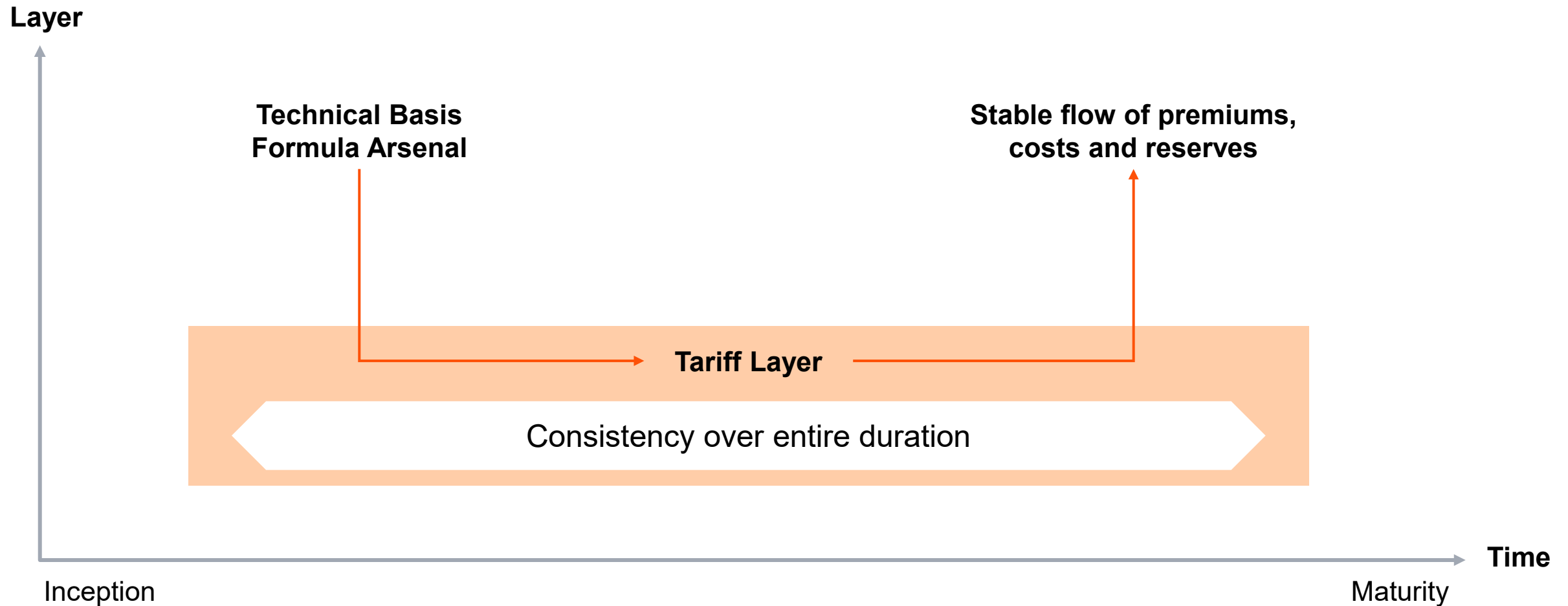


Table 1. Life table for the total population: United States, 1999

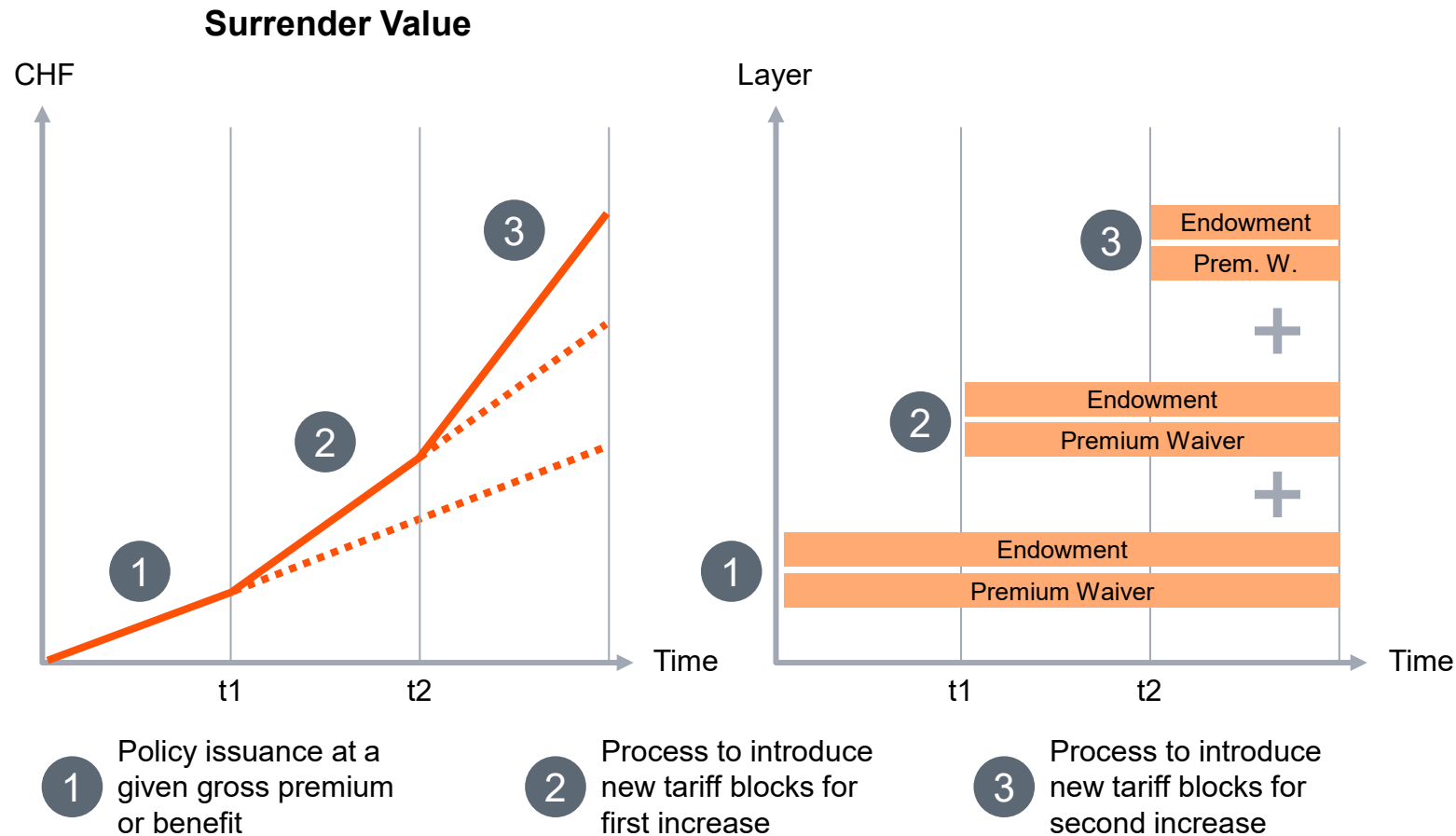
Age	Number of lives at age x, \$l_x\$	Number of survivors to age \$x+n\$, \$l_{x+n}\$	Number of deaths between ages \$x\$ and \$x+n\$, \$d_x\$	Number of deaths between ages \$x\$ and \$x+n\$, \$D_x\$	Number of deaths between ages \$x\$ and \$x+n\$, \$C_x\$	Number of deaths between ages \$x\$ and \$x+n\$, \$N_x\$	Number of deaths between ages \$x\$ and \$x+n\$, \$M_x\$
00	100000	95720	4280	100000	1132	2774172	12674
01	95720	91440	4280	95720	94	2674172	11542
02	91440	87160	4280	91440	65	2574172	11448
03	87160	82880	4280	87160	47	2485839	11383
04	82880	78600	4280	82880	40	2396206	11336
05	78600	74320	4280	78600	37	2309441	11296
06	74320	70040	4280	74320	34	2225447	11259
07	70040	65760	4280	70040	31	2144134	11225
08	65760	61480	4280	65760	28	2065415	11194
09	61480	57200	4280	61480	25	1989205	11166
10	57200	52920	4280	57200	23	1915422	11141

	d_x	D_x	C_x	N_x	M_x
00	1169	100000	1132	2774172	12674
01	100	95720	94	2674172	11542
02	72	92613	65	2574172	11448
03	53	89633	47	2485839	11383
04	47	86765	40	2396206	11336
05	45	83994	37	2309441	11296
06	43	81313	34	2225447	11259
07	40	78719	31	2144134	11225
08	37	76210	28	2065415	11194
09	35	73783	25	1989205	11166
10	33	71435	23	1915422	11141

Each individual tariff can be represented as an independent layer



Let's increase the premium on our policy

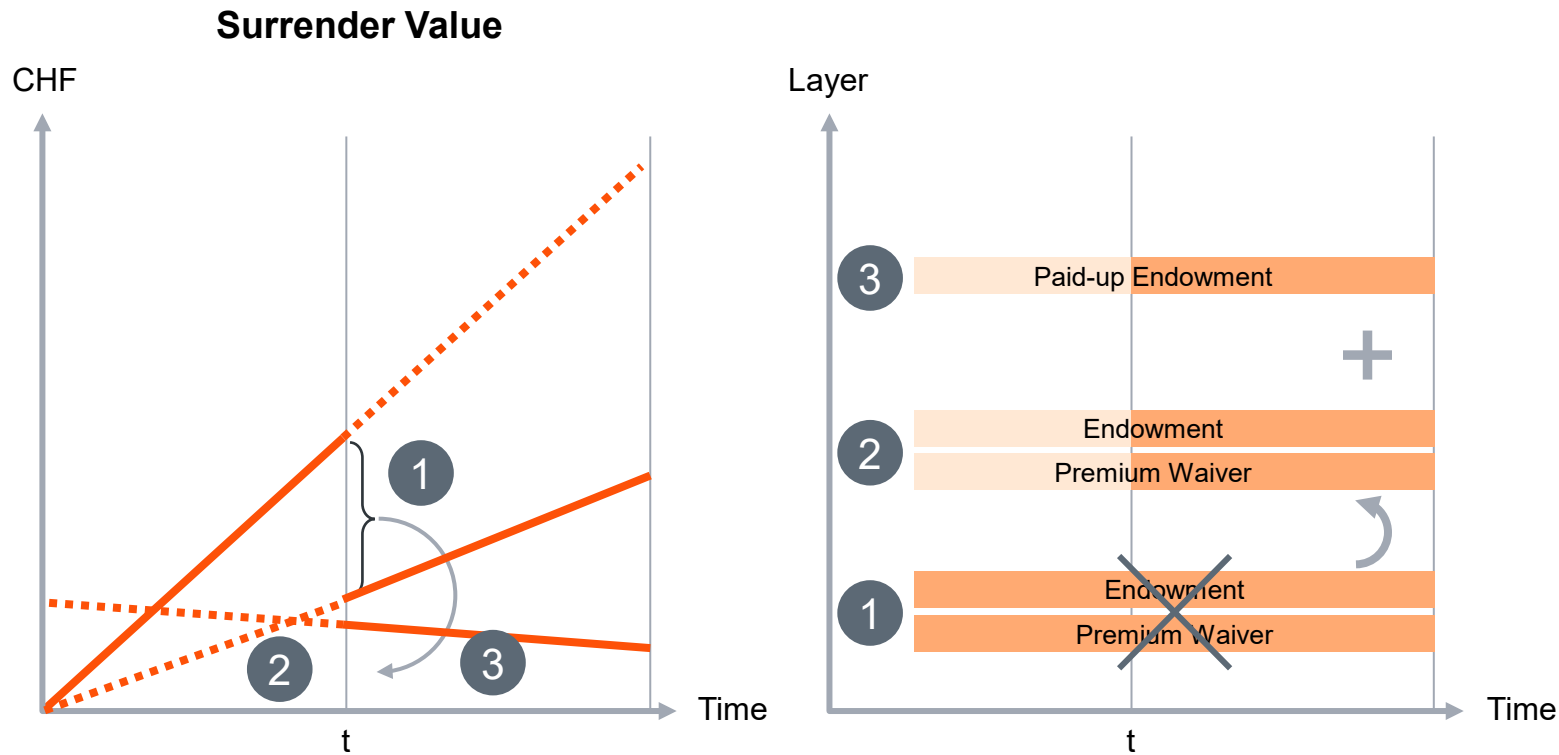


Typical Use cases

1. Automatic increase, for example
 - a. Inflation linked
 - b. 3a maximum cap
2. Permitted increase within same policy

- + High Transparency**
Each actuarial value can be traced with high precision in the system.
- + Easy match for sales commissions**
Allow multi agent/brokers per policy.
- Difficulty with rebate structures**
Cumbersome to reward policyholders that increase premiums.
- Handling of spontaneous premiums**
Generate additional layers with separate tariffication. Unclear how to reward policyholders.

Let's reduce the premium on our policy (Partial Conversion)



1 Determine proportion of reduction to be performed on existing tariff layer to reach target premium or benefit.

2 Process to recalculate new tariff blocks and reperform all actuarial calculations.

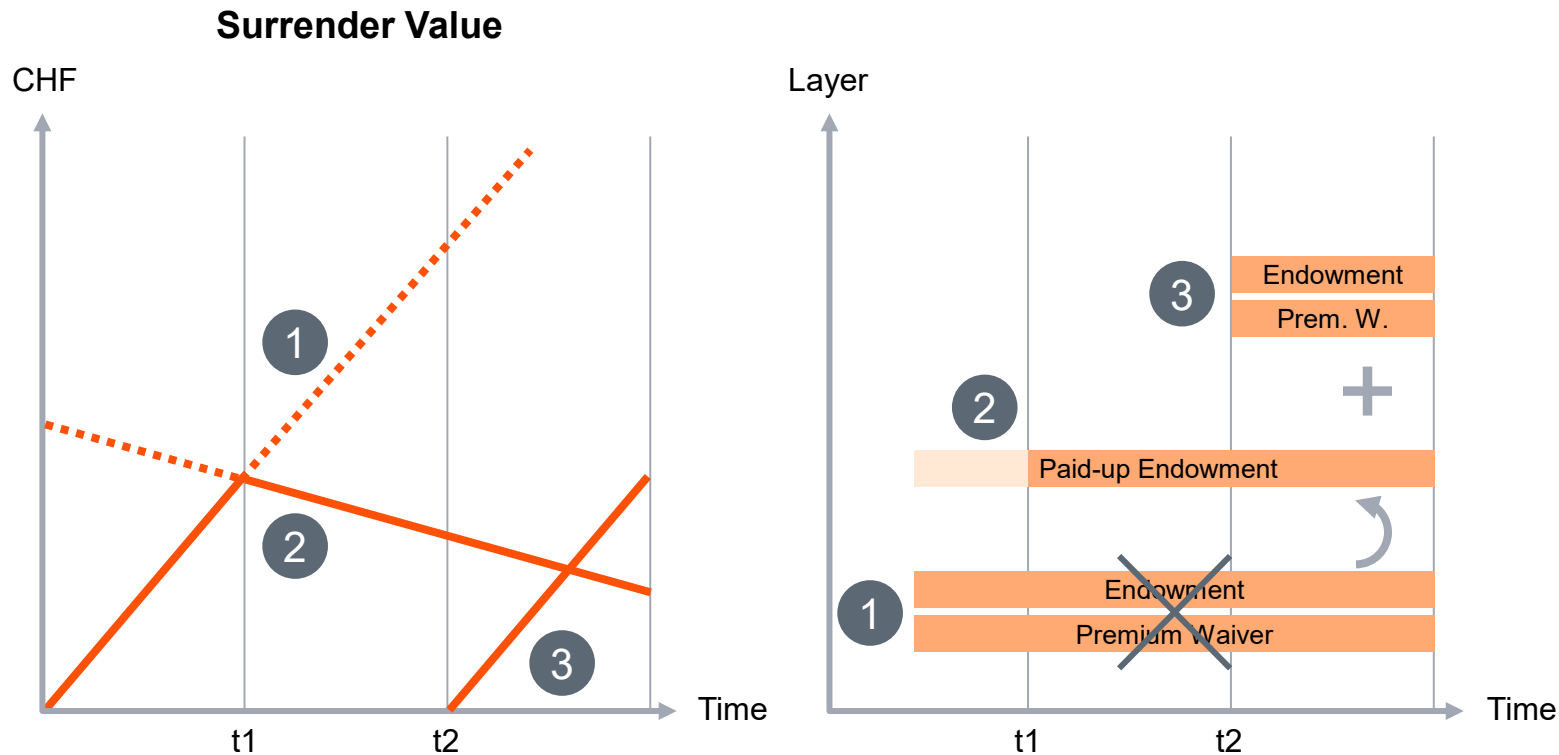
3 Process to add new block to model the amount parked from the simulated partial surrender. Recalculate all actuarial fields on converted layer.

Typical Use cases

1. Policyholder exerts legal right to reduce regular premium payments at any given time in the year
2. Policyholder protection needs have decreased

- **Complexity**
Requires multiple subsequent processes exacerbated if the policy already had multiple operations.
- **Detriment to the policyholder**
Leads to forceful amortization of acquisition costs for lack of alternative.
- **Database pollution**
Transparency becomes limited for policies with frequent operations.

Let's suspend the premium on our policy



- 1 Process to capture suspension and prematurely end the existing tariff layers.
- 2 Process to open a conversion layer to park the accumulated surrender value
- 3 Process to open additional regular premium Tariffs layers that reach the initial premium amount. Unclear if a new underwriting process is required and what tariff generation should be used.

Typical Use cases

1. Retain customer by avoiding a lapse or a conversion of the policy
2. Policyholder has temporary financial difficulties

- **Complexity**
 Requires multiple subsequent processes exacerbated if the policy already had multiple operations
- **Detriment to the policyholder**
 Lapse penalties are applied even though acquisition costs are levied once more in step 3.
- **High Maintenance**
 Existing system business rules might need to be overridden, leading usually to a manual process.

Commutation numbers come with a set of limitations



Advantages from calculating with commutation numbers

- Simple products can be modelled easily
- The meaning of commutation numbers is easy to interpret
- Can be hard coded and uploaded to IT-systems



But there are some limits

- The number of formulas to create grows quickly with the number of products and product complexity
- Higher maintenance efforts to manage the formulas
- Reduced flexibility to introduce new products and manage post issuance operations
- Each policy operation multiplies the number of tariff layers to be maintained

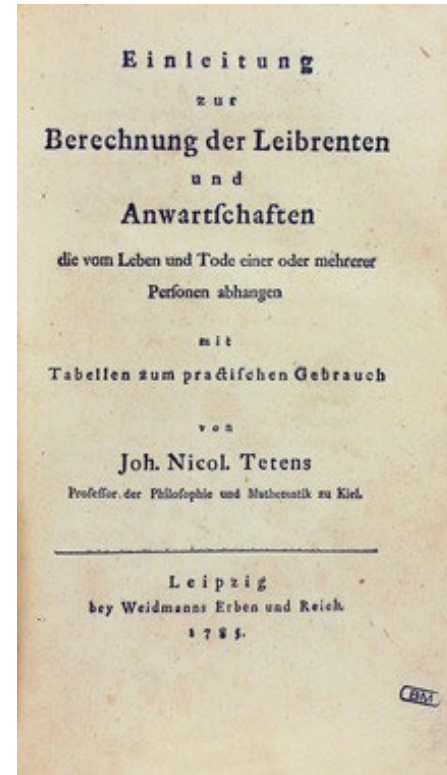
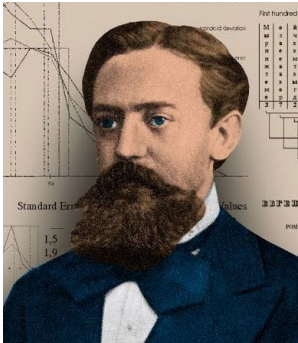


Table 1. Life table for the total population United States, 1999

Age	Number of lives at age x	Number dying between age x and x+1	Number surviving to age x+1	Number of years lived between age x and x+1	Number of years lived beyond age x	Number of years lived beyond age x, weighted by the number of lives at age x
0	100000	1169	98831	1132	2774172	12674
1	98831	100	98731	94	2674172	11542
2	98731	72	98659	65	2578452	11448
3	98659	53	98606	47	2485839	11383
4	98606	47	98559	40	2396206	11336
5	98559	45	98514	37	2309441	11296
6	98514	43	98471	34	2225447	11259
7	98471	40	98431	31	2144134	11225
8	98431	37	98394	28	2065415	11194
9	98394	35	98359	25	1989205	11166
10	98359	33	98326	23	1915422	11141

	d_x	D_x	C_x	N_x	M_x		
00	1169	100000	1132	2774172	12674		
31	100	95720	94	2674172	11542		
31	72	92613	65	2578452	11448		
59	53	89633	47	2485839	11383		
06	47	86765	40	2396206	11336		
59	45	83994	37	2309441	11296		
14	43	81313	34	2225447	11259		
71	40	78719	31	2144134	11225		
8	0.000379	98431	37	76210	28	2065415	11194
9	0.000352	98394	35	73783	25	1989205	11166
10	0.000334	98359	33	71435	23	1915422	11141

Actuarial Science was a century ahead of the developments in Information Technology



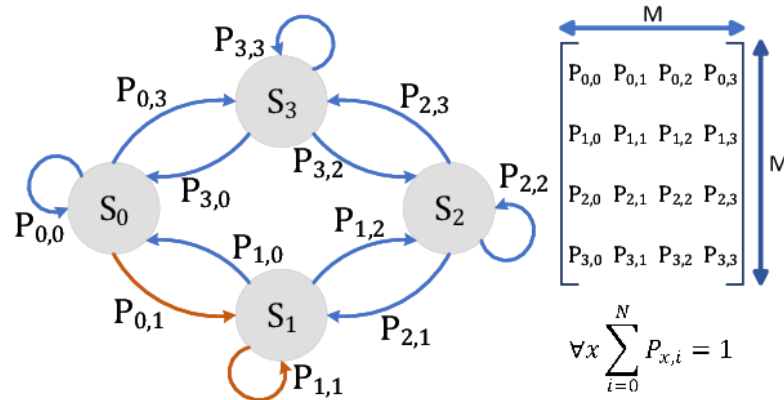
Andrei Andrejewitsch Markow
1856 - 1922



Thorvald Nicolai Thiele
1838 - 1910

Key Points in Brief

- The model describes the possible transition from one contract state to the next, **regardless of potential contract changes**.
- For each time step, a **transition probability** is defined, based on the assumptions of the relevant tariff.
- Using the Markov chain (and with the help of the Thiele recursion), **commutation-based formulas can be rewritten recursively**.
- The key is the **use of vectors** for benefits, gross premiums, and costs in calculating the actuarial reserve.

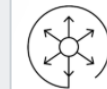


A new Standard is being established

Leading Software providers are shifting

A different way of expressing commutation numbers

- First use in the 90s, but still too expensive to represent a cost-effective improvement
- 2010 core systems start to reach sufficient performance



Flexibles Semi-Markov-Modell zur Berechnung von Barwerten, Beiträgen und Reserven



Markov-Ansatz

Die versicherungsmathematische Basis reduziert sich mit diesem Ansatz auf eine performante Formel für alle Produkte. Damit einher geht eine deutliche Vereinfachung der Tariflandschaft. Zieltarife für Migrationen lassen sich ohne Abweichung zum Altsystem modellieren.

Intuition of the state-model using Markov Chains

Model Assumptions

Z = State Set {a,b,c,d}

A = Active

D = Dead

I = Disabled

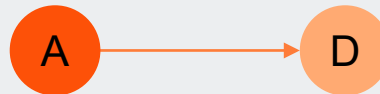
R = Reactivated

- Model describes the possible Transition from one state to the next
- For each time-step a transition probability is defined

Examples from Life Insurance

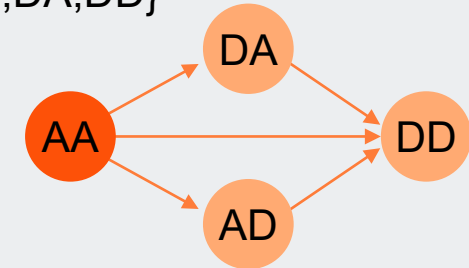
Term Insurance

Z = {A,D}



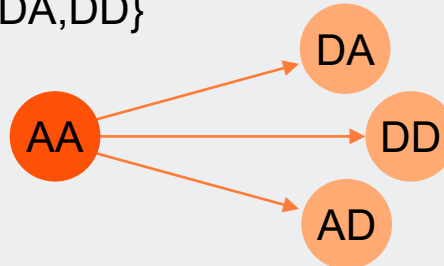
Annuity on 2 Lives

Z = {AA,AD,DA,DD}



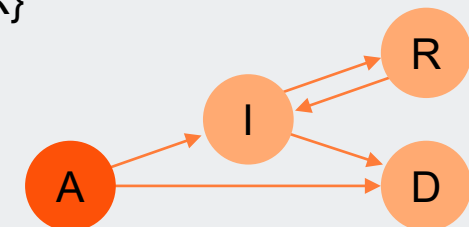
Term Insurance on 2 Lives

Z = {AA,AD,DA,DD}



Disability Annuity

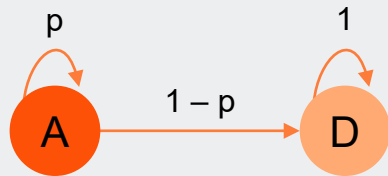
Z = {A,I,D,R}



Let's take the simplest example

Term Life on 1 Life

$$Z = \{A, D\}$$



Probability per time-step:

$$P(A \rightarrow A) = p$$

$$P(A \rightarrow D) = 1 - p$$

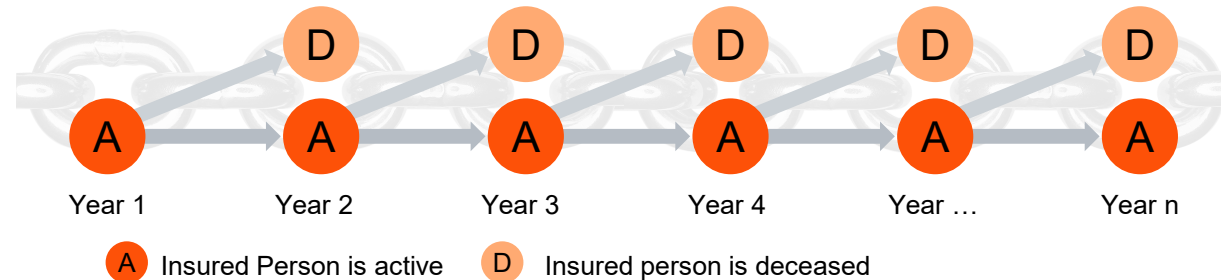
$$P(D \rightarrow D) = 1$$

Number of Actives and Dead

$$a(n) = p \times a(n-1)$$

$$d(n) = (1 - p) \times a(n-1) + d(n-1)$$

When viewed as a process over discrete time steps, the chain becomes apparent



Or expressed in a Matrix-Form:

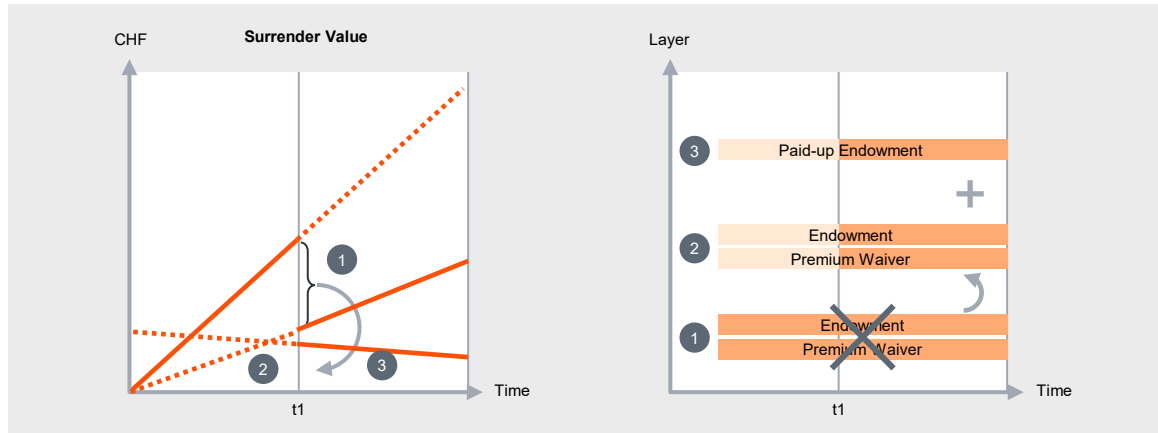
$$\begin{pmatrix} a(n) \\ d(n) \end{pmatrix} = \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdot \begin{pmatrix} a(n-1) \\ d(n-1) \end{pmatrix} = \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdot \dots \cdot \begin{pmatrix} p & 0 \\ 1-p & 1 \end{pmatrix} \cdot \begin{pmatrix} a(0) \\ d(0) \end{pmatrix} = \begin{pmatrix} p^n & 0 \\ 1-p^n & 1 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ 0 \end{pmatrix}$$

And that's where the **Thiele Recursion** makes an entrance:

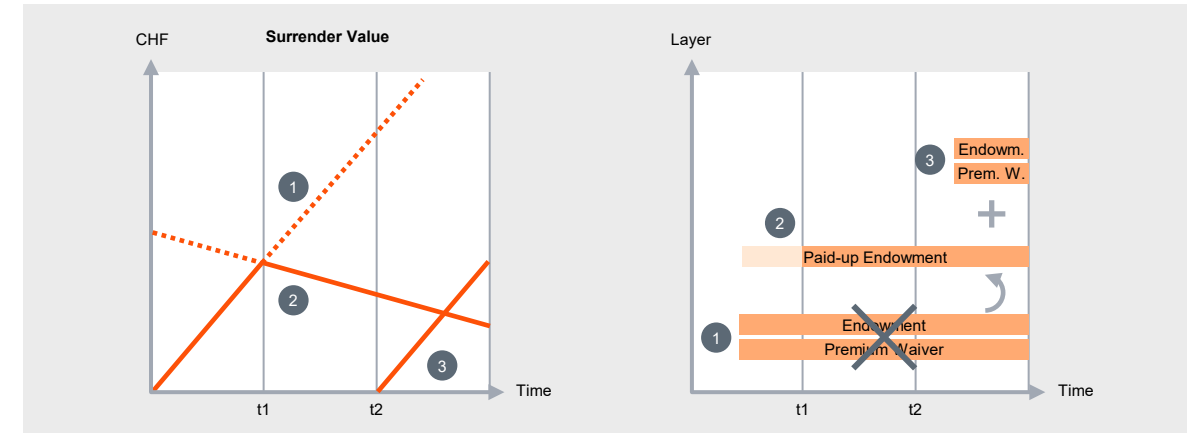
$$V_k(t) = \underbrace{b_k(t)}_{\text{Costs - Gross Premium}} + \underbrace{v(t, t^+) \times \sum_{i \neq k} p_{k,i}(t^+)}_{\text{Transition probability from state k to state i}} \left(\underbrace{b_{k,i}(t^+) + V_i(t^+)}_{\text{Benefit in case of state change}} \right) + \underbrace{v(t, t^+) \times \left(1 - \sum_{i \neq k} p_{k,i}(t^+) \right)}_{\text{interest from t to t^+}} \times V_k(t^+)$$

Side by side – Suddenly it's all just about updating vector trajectory

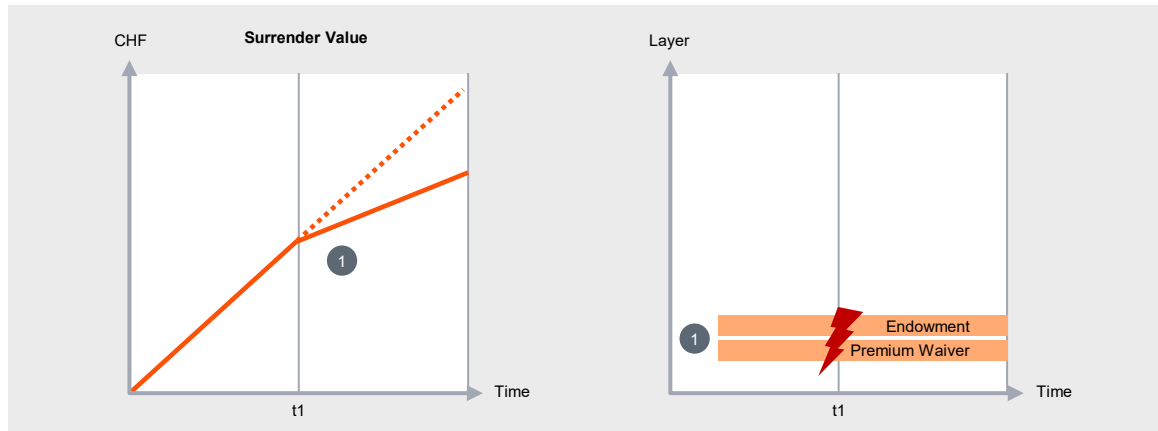
Reduction with commutation numbers



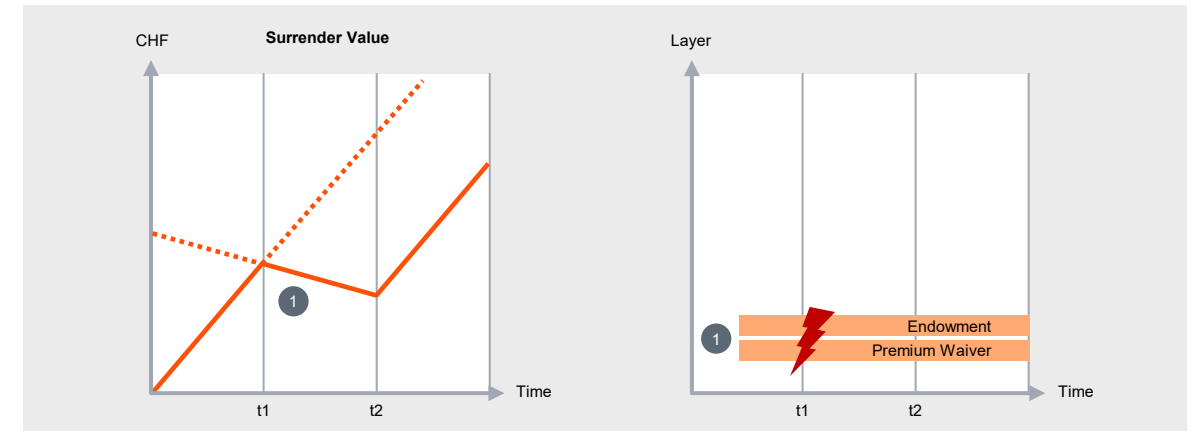
Suspension with commutation numbers



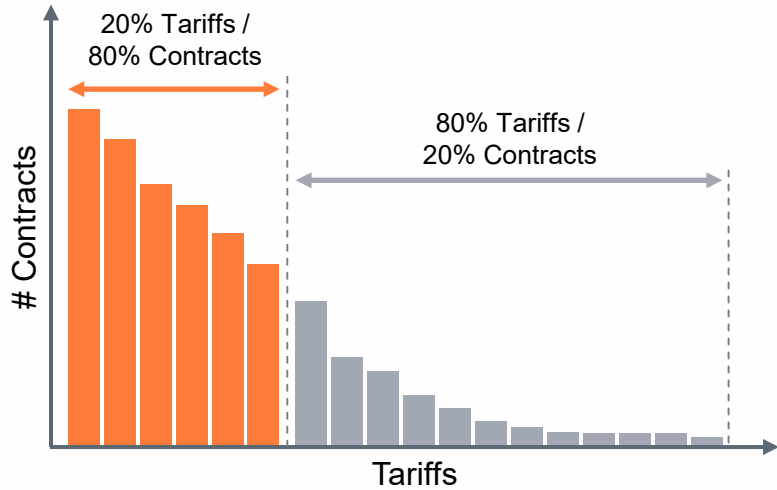
Reduction with Markov approach



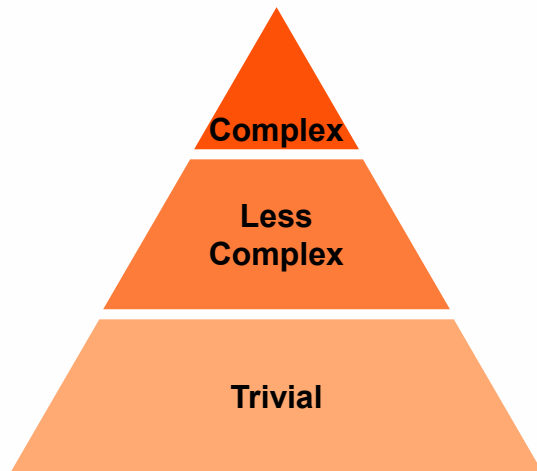
Suspension with Markov approach



„Long-Tail-Problem“ of Life portfolios



Tariff-Complexity



And now there is a ready toolbox for portfolio rationalization

1

Post extra Reserve – One-time contribution at migration

$$V \pm V^{MIG} = -B \cdot \ddot{a}_{x+t,n-t} + A_{x+t,n-t} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{x+t,n-t} + \gamma_1 \cdot T \cdot \ddot{a}_{x+t,n-t}$$

2

Increase Policyholder Benefit

$$V = -B \cdot \ddot{a}_{x+t,n-t} + A_{x+t,n-t} \pm \hat{A}_{x+t,n-t} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{x+t,n-t} + \gamma_1 \cdot T \cdot \ddot{a}_{x+t,n-t}$$

3

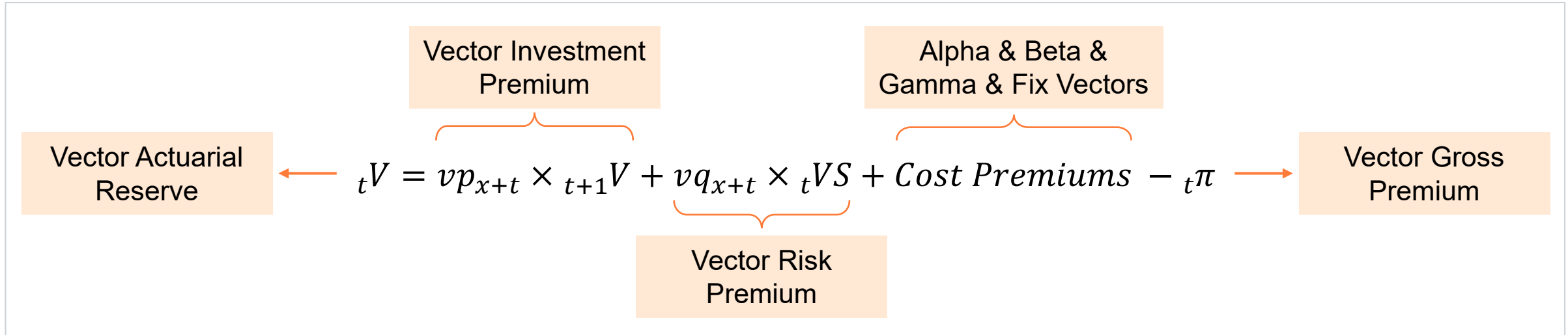
Migration Premium / Migration Discount

$$V = -B \cdot \ddot{a}_{x+t,n-t} \pm MIGBTG \cdot \ddot{a}_{x+t,n-t} + A_{x+t,n-t} + \alpha \cdot \min(n-t, 35-t) \cdot B + \beta \cdot B \cdot \ddot{a}_{x+t,n-t} + \gamma_1 \cdot T \cdot \ddot{a}_{x+t,n-t}$$



- Migrate full vectors of benefits , premiums and costs
- No impact on policyholders
- Post migration reserve instead

The Thiele recursion brings most actuarial calculations in one formula



Product Flexibility

New Product Design possibilities as it becomes possible to offer non-linear benefits and premiums, while enabling self service possibilities.

International relevance as the model is neutral to Swiss specific requirements and goes back to scientific fundamentals. One formula for all Tariffs.

Process Streamlining

Need fewer resources to define requirements, maintain and update the system and implement legal changes, due to single formula shared by all operations.

Faster execution of operations, as less manual steps are needed, due to the ability to influence calculations directly via the recursive vectors.

Migration Simplification

Ensure Regulatory Compliance of guaranteed benefits and premiums due to vectorial approach that can also be migrated from legacy.

Faster Migration as we can rationalize and reduce the amount of Tariffs to be migrated, due to the ability of merging individual tariff blocks with the recursion.

The theory looks simple, but
what about real examples ?

Demo



A single Markov
chain has the
potential of
handling the entire
life portfolio



Markov Models in life insurance, the key take-aways

1

Simplification of Processes

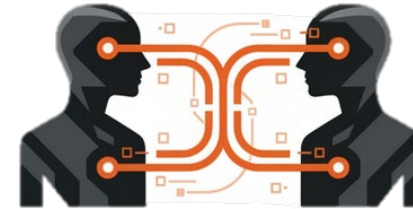
2

Cut tariff formula maintenance

3

Game changer for system migration

Next Developments



September 2026

- Full agent to agent Integration PoC
- AWS cloud deployment



November 2026

- Paper Publication in Tokyo
- Full GitHub release